Abstract
For RAN ships operating in shallow water, it is important to understand the phenomena of squat and wave-induced motions so as to avoid grounding. The British Admiralty Manual of Navigation gives guidelines to predict squat allowances in different conditions. In this article, we discuss current research in this field, and highlight the important phenomenon of transcritical squat and its implications for frigate- and destroyer-type ships.

What is squat?
Squat is a ship’s tendency to sit lower in the water as it travels faster. This is caused by the Bernoulli effect. Water is accelerated along the sides of and beneath the hull, and the free surface must drop downwards to maintain energy conservation. The ship essentially makes its own wave trough in which it sits, as shown in Figure 1. This effect is additional to the trailing wave pattern with which we are all familiar.

Figure 1: The squat effect. (Top) Ship at rest. (Bottom) Ship under way.

Squat comprises a midship sinkage and a change in trim. High block-coefficient ships such as tankers and bulk carriers tend to trim down by the bow. For medium block-coefficient ships such as containerships and many warships, changes in trim are more subtle, and are sensitive to the exact hull shape.

Ideally, a captain should study and understand each ship’s trim behaviour with increasing speed. This may be done with a trim inclinometer, or in the old days, marbles on the deck. For example, the captain of one of the containerships on which we did field trials in Hong Kong knew that his ship had a strong tendency to trim by the bow when under way. He therefore loaded his ship by the stern, so that it would have close to level trim when under way.

How do we predict squat?
The new PIANC guidelines for approach channel design (PIANC 2014) give a good overview of the state-of-the-art methods in ship squat prediction, and contain several formulae to predict squat. These formulae have mostly been developed for cargo ships, so care should
be taken when using them for very slender warships. One simple formula which should be fairly accurate for frigates and destroyers in open water at low speeds is the Tuck formula:

$$s_{\text{midships}} = c_s \frac{\nabla}{L_{PP}^2} \frac{F_h^2}{\sqrt{1 - F_h^2}}$$

For frigates and destroyers, the sinkage coefficient is around $c_s = 2.0$. As an example using SI units, let us consider a generic frigate with length between perpendicu- lars $L_{PP} = 125$m and displacement $\nabla = 4000$m$^3$. It is travelling in water depth (including tide) of $h = 11.0$m, at a speed of 12 knots, i.e. $U = 6.17$m/s. The depth-based Froude number is then $F_h = \frac{U}{\sqrt{gh}} = 0.594$. Here $g$ is the acceleration due to gravity (9.81m/s$^2$).

Using the Tuck formula, the midship sinkage would be 0.22m in this case. This is very small! By comparison, bulk carriers and containerships often have bow sinkage in excess of 1 metre in shallow water, so that squat is a controlling factor. Cargo ships have large squat because of their large beam-to-length ratios. Frigates, having very low beam-to-length ratios, tend to have small squat at similar speeds.

Is that the end of the story?

Not quite. Cargo ships have very limited power, and are unable to travel at high speeds in shallow water because of the very high resistance this entails. However frigates and destroyers are able to, and may have to, travel at high speeds in shallow water. We can see from the Tuck formula above that strange things start to happen when the depth-based Froude number approaches 1. This speed is known as the “critical speed” ($F_h = 1.0$).

The original Tuck formula is only valid at low speeds ($F_h < 0.7$) or very high speeds ($F_h > 1.2$). The region in between is known as the “transcritical” region, in which a different formula must be used.

The critical speed

The critical speed in shallow water ($F_h = 1.0$) is where the long waves produced by the ship travel at the same speed as the ship, so there is almost no restoring force on the free surface. Therefore large waves are produced, accompanied by large wave resistance and large changes in sinkage and trim.

The critical speed is shown in Table 1 for various water depths.

<table>
<thead>
<tr>
<th>Water depth including tide, metres</th>
<th>Critical speed, metres/sec</th>
<th>Critical speed, knots</th>
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<tbody>
<tr>
<td>10</td>
<td>9.9</td>
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<td>18</td>
<td>13.3</td>
<td>25.8</td>
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<tr>
<td>20</td>
<td>14.0</td>
<td>27.2</td>
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</table>

Table 1: Shallow-water critical speed
As well as large squat, the critical speed is also accompanied by very large resistance, as discussed in Gourlay (2012).

**Measured ship squat at transcritical speeds**

Probably the best model test data on warship squat at transcritical speeds was that of Graff et al. (1964). They tested a 3.0m model of a Taylor standard series A3 hull, which is a frigate-type hull, but with smaller displacement than modern frigates. When scaled at 21:1 to typical frigate length, the measured stern sinkage is shown in Figure 2.

![Figure 2: Measured stern sinkage of a Taylor A3 frigate-type hull, at 21:1 scale with $L_{PP} = 126m$ and displacement $3400m^3$, in water of depth $15.25m$. Critical speed = 24.2 knots. Results from Graff et al. (1964).](image)

In this case we see that, at just below the critical speed, the stern sinkage reaches 2.9 metres. This is caused partly by a large midship sinkage, and partly by a large stern-down trim. Bearing in mind that the maximum draft of frigates and destroyers tends to occur at the stern, this large stern sinkage is of particular concern when travelling at transcritical speeds in shallow water.

No published data appears to exist on transcritical squat of warships at full scale. However with advances in technology, such measurements are now entirely feasible, and have been done for many years on cargo ships at low speeds, see for example Gourlay & Klaka (2007).

The most striking example of ship grounding due to transcritical squat is that of the QE2 in Vineyard Sound, Massachusetts (UKMAIB 1993).

**Predicting transcritical ship squat**

A theoretical formulation to predict flow patterns and ship squat at transcritical speeds was developed by Gourlay and Tuck (2001). This theory has been incorporated into CMST’s “ShallowFlow” software. Figure 3 shows the calculated wave pattern produced by a frigate travelling at 21 knots in 12m water depth, i.e. at the critical speed. We can see the large waves that are produced, with crests almost perpendicular to the ship’s track.
Figure 3: Wave pattern for a frigate travelling at 21 knots in 12m water depth. Colours show free surface height in metres above still water level. Calculated using ShallowFlow software.

A simple semi-empirical formula for maximum squat through the transcritical range was given in Gourlay (2006). This gives

\[ s_{\text{max, stern}} \approx 1.5 \frac{V}{L_{PP} h} \]

As an example, a generic frigate with \( L_{PP} = 125 \text{m} \) and \( V = 4000 \text{m}^3 \), travelling in water of depth 11.0m, would have a maximum stern sinkage of around 4.4m at just beneath the critical speed of 20.2 knots. With a stern draft of 7.0m, this stern sinkage would cause the ship to run aground.

**Wave-induced motions**

For operations in shallow ocean areas, long-period swells cause heave, pitch and roll of the ship, which can lead to grounding. An example for cargo ships is the MV Capella Voyager grounding in long-period swells at Whangarei, New Zealand, despite having 2.6m static under-keel clearance (MNZ 2003).
Australian research in the field

Australia has a long history of research into ship hydrodynamics in shallow water, including:

- John Michell (University of Melbourne) laid the foundations for the thin-ship theory of ship hydrodynamics in deep and shallow water (Michell 1898), and developed the thin-ship wave resistance formula which is still used internationally today
- Ernie Tuck (University of Adelaide) developed the Tuck theoretical and empirical formulae for ship squat (Tuck 1966), which form the basis of most modern squat prediction methods, as given in PIANC (2014)
- The Centre for Marine Science and Technology at Curtin University (Perth) develops “ShallowFlow” software for ship squat, undertakes full-scale trials and develops UKC guidelines for ports
- Australian Maritime College (Launceston) undertakes model testing and has a large database of ship squat measured at model scale
- OMC International (Melbourne) develops “DUKC” software which determines safe tidal windows for many ports around the world, including the effects of squat and wave-induced motions
- Cyberiad (Adelaide) develops “Flotilla” software which calculates ship resistance and squat in deep and shallow water

Conclusions

Warships such as frigates and destroyers have slender hulls, and hence have minimal squat at low speeds. However, their efficient hull shapes and high power means that they are able to travel at speeds close to the shallow-water critical speed. This speed depends on the water depth. Near the critical speed, resistance, wave patterns and sinkage are very large. Stern sinkages in the order of 4-5m are possible for frigates and destroyers near the critical speed, with associated grounding risk. In swell conditions, heave, pitch and roll of the ship should also be taken into account in order to avoid grounding.

Symbols used

- \( c_s \): sinkage coefficient
- \( g \): acceleration due to gravity = 9.81 m/s\(^2\)
- \( h \): water depth (m)
- \( F_H \): depth-based Froude number \( U / \sqrt{gh} \)
- \( L_{PP} \): length between perpendiculars (m)
- \( s_{\text{max, stern}} \): maximum stern sinkage (m) through the transcritical speed range, positive downward
- \( s_{\text{midships}} \): sinkage (m) at midships, positive downward
- \( U \): ship speed (m/s)
- \( \nabla \): volume displacement (m\(^3\))

References


