# COMPARISON OF WAMIT WITH MEASURED WAVE-INDUCED MOTIONS OF CONTAINER SHIPS AND BULK CARRIERS, USING AN ADJUSTMENT FOR SHIP SPEED

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**SUMMARY**

A computationally-efficient method is presented for approximating wave-induced motions of ships at low Froude number, using zero-speed Green functions as calculated using WAMIT software. The method is not new: other panel codes such as HydroSTAR and VERES3D offer a low-Froude-number approximation based on zero-speed Green functions. This article aims to set out mathematical equations for a speed-adjusted method, and validate the results against a range of available model test data. The method may be used in shallow or deep water.

**NOMENCLATURE**

SI units are used throughout.

| | |
|---|---|
| AP | Aft perpendicular |
| CoG | Ship centre of gravity |
| DTC | Duisburg Test Case container ship |
| GM | Transverse metacentric height above CoG |
| FP | Forward perpendicular |
| KG | CoG height above keel |
| LBP | Length between perpendiculars |
| LCG | Longitudinal centre of gravity |
| LOA | Length overall |

| | |
|---|---|
| $A_{jk}$ | Hydrodynamic added mass matrix |
| $B_{jk}$ | Hydrodynamic damping matrix |
| $C_{jk}$ | Hydrostatic restoring coefficient matrix |
| $c_B$ | Block coefficient |
| $k$ | Wave number $2\pi/\lambda$ |
| $k_{xx}$ | Roll gyradius |
| $k_{yy}$ | Pitch gyradius |
| $k_{zz}$ | Yaw gyradius |
| $M_{jk}$ | Mass matrix |
| $U$ | Ship speed |
| $X_j$ | Wave load complex amplitude |
| $z_a$ | Heave amplitude at aft perpendicular, defined as half of peak-to-peak heave |
| $z_f$ | Heave amplitude at forward perpendicular, defined as half of peak-to-peak heave |
| $\varepsilon_a$ | Heave phase at aft perpendicular (deg), ahead of wave elevation at CoG |
| $\varepsilon_f$ | Heave phase at forward perpendicular (deg), ahead of wave elevation at CoG |
| $\zeta$ | Wave amplitude, defined as half of wave height |
| $\eta$ | Wave elevation at ship CoG |
| $\lambda$ | Wavelength |
| $\beta$ | Wave heading, anticlockwise from stern-on |
| $\omega_0$ | Wave frequency (rad/s) |
| $\omega_e$ | Encounter frequency (rad/s) |
| $\xi_k$ | Complex ship motion amplitude |

## 1 INTRODUCTION

WAMIT is a panel method for calculating wave-induced loads and motions of ships and offshore structures (WAMIT 2023). WAMIT uses zero-speed Green functions and does not have an allowance for ship forward speed. A version called TIMIT using forward-speed Green functions was developed in the 1990s (Korsmeyer et al. 1999), but is not generally available.

## 2 ZERO-SPEED SHIP MOTIONS IN WAMIT

The ship origin, as used in WAMIT, is the ship centreline, waterline and LCG. The degrees of freedom are shown in Table 1.

**Table 1. Ship motion degrees of freedom**

| | |
|---|---|
| $x_1$ | Surge (positive forwards) |
| $x_2$ | Sway (positive to port) |
| $x_3$ | Heave (positive upwards) |
| $x_4$ | Roll (positive to starboard) |
| $x_5$ | Pitch (positive bow-down) |
| $x_6$ | Yaw (positive bow-to-port) |

We first consider a stationary ship in regular waves (long-crested waves with a single frequency). Wave phasing is defined such that, at the ship CoG,

$$\eta = \zeta \cos(\omega_0 t) \tag{1}$$

The equation of ship motion may be written (Salvesen et al. 1970, eq. 1) as

$$\sum_{k=1}^{6}\left[M_{jk} + A_{jk}\right]\ddot{x}_k + \sum_{k=1}^{6} B_{jk}\dot{x}_k + \sum_{k=1}^{6} C_{jk}x_k = Re\left\{X_j e^{i\omega_0 t}\right\} \; ; \; j = 1..6 \tag{2}$$

In WAMIT, the terms are calculated as follows:

- $M_{jk}$ is the 6x6 generalized mass matrix, calculated as in WAMIT (2023, eq. 3.3)
- $A_{jk}$, $B_{jk}$ are the 6x6 added mass and damping matrices. These depend on the *oscillation frequency*, which for a stationary ship is the same as the wave frequency. $A_{jk}$, $B_{jk}$ are calculated by oscillating the ship in calm water and calculating the force in phase with the displacement (giving added mass) and the force in phase with the velocity (giving damping). This is the "radiation problem", as it concerns waves radiated from the ship due to its oscillations.
- $C_{jk}$ is the 6x6 hydrostatic restoring matrix (WAMIT 2023, p3-3)
- $X_j e^{i\omega_0 t}$ are the wave exciting forces, with $X_j$ the complex amplitude. These are a function of wave frequency and wave heading, and are solved by keeping the ship vertically and horizontally fixed in its calm-water position, and calculating the wave-induced pressures and forces on the ship. This is called the "diffraction problem", as it concerns waves diffracting around the hull.

Equation (2) is a typical spring-mass-damper equation, for which the motion oscillates at the same frequency as the forcing frequency. The motion can be written

$$x_k = Re\{\xi_k e^{i\omega_0 t}\} \tag{3}$$

Here $\xi_k$ are the complex motion amplitudes. By substituting equation (3) into equation (2), we find that the complex motion amplitudes are the solution (WAMIT 2023, p3-5) of the matrix equation

$$\sum_{k=1}^{6}\left[-\omega_0^2\left(M_{jk} + A_{jk}\right) + i\omega B_{jk} + C_{jk}\right]\xi_k \; = X_j \; ; \;\; j = 1..6 \tag{4}$$

This gives the solution to the motion amplitudes and phases, for a ship at zero speed in regular waves at a given wave heading and frequency. For an irregular seaway, the motions may be linearly superposed for all headings and frequencies (St Denis and Pierson 1953).

## 3 USING WAMIT WITH A FORWARD-SPEED ADJUSTMENT

In this study, WAMIT will be used to approximate the wave-induced motions of a ship at forward speed. The method proceeds as follows.

Equation (2) as developed in Salvesen et al. (1970) is valid for ships at forward speed. In that case, the wave loading and the ship response both oscillate at the *encounter frequency* (Salvesen et al. 1970, eq. 22)

$$\omega_e = \omega_0 - kU\cos\beta \tag{5}$$

Thus, the encounter frequency may be calculated for any input ship speed, heading and wave frequency.

The added mass and damping are calculated with WAMIT at the correct *encounter frequency*, as this is the frequency at which the ship is oscillating.

The wave loads are calculated with WAMIT at the correct *wave frequency*. The primary component of wave loading is the Froude-Kriloff component, which is the hydrodynamic pressure integrated over the ship's hull, if the ship did not disturb the wave. This is very sensitive to the wavelength, as there are humps and hollows depending on the ratio of wavelength to ship length. Fig. 17 of Gourlay et al. (2015) shows that wave loads vary markedly with wave frequency, with only a small effect of ship speed. Therefore, to ensure the wavelength and Froude-Kriloff component are correct, we use the wave frequency for the wave load calculations.

A MATLAB program is written to take in the WAMIT-calculated coefficients and solve the matrix equation (4).

Some error is expected in the above approximations, as the added mass, damping and wave loads are calculated using zero-speed Green functions. However, as it is intended to apply the method to large ships at low Froude numbers, this effect may be small.

The following sections illustrate calculations using the proposed method, and comparison with model tests. In all plotted results, lines are numerical calculations and dots are experimental results. Different colours represent different speeds. In some cases, experimental results are available at different speeds; in other cases, numerical results are given at different speeds to show the effect of speed on the results. The "0 knots" results are calculated using the described method with $U = 0$; these results match the zero-speed RAOs output by WAMIT.

# 4 RESULTS FOR A 190 M BULK CARRIER

We consider the model test case of the Panamax bulk carrier “Ship G” which was tested at 8 and 10 knots in head seas at Flanders Hydraulics (Vantorre & Journée 2003). Test conditions are shown in Table 2.

**Table 2. 190 m bulk carrier model test conditions**

| Parameter | Value | Source |
|---|---|---|
| LOA | 190.00 m | Vantorre & Journée (2003, Table 1) |
| LBP | 180.00 m | " |
| Beam | 33.00 m | " |
| Maximum draft | 13.00 m | " |
| $c_B$ | 0.85 | " |
| Tested draft | 11.6 m | Vantorre & Journée (2003, Table 3) |
| KG | 11.6 m | " |
| Water depth | 13.6 m | " |
| $k_{yy}$ | 25% LBP | " |
| $k_{xx}$ | 44.8% Beam | Gourlay et al. (2015) |

A 4384-panel surface mesh for Ship G, up to the modelled waterline, was developed with OCTOPUS as shown in Gourlay et al. (2015, Fig. 6). This hull mesh was modelled with WAMIT v7.5, using the inputs shown in Table 2. Heave and pitch motions were calculated at 0, 8, 10 knots using the method described in Section 3. Results are shown in Figure 1.

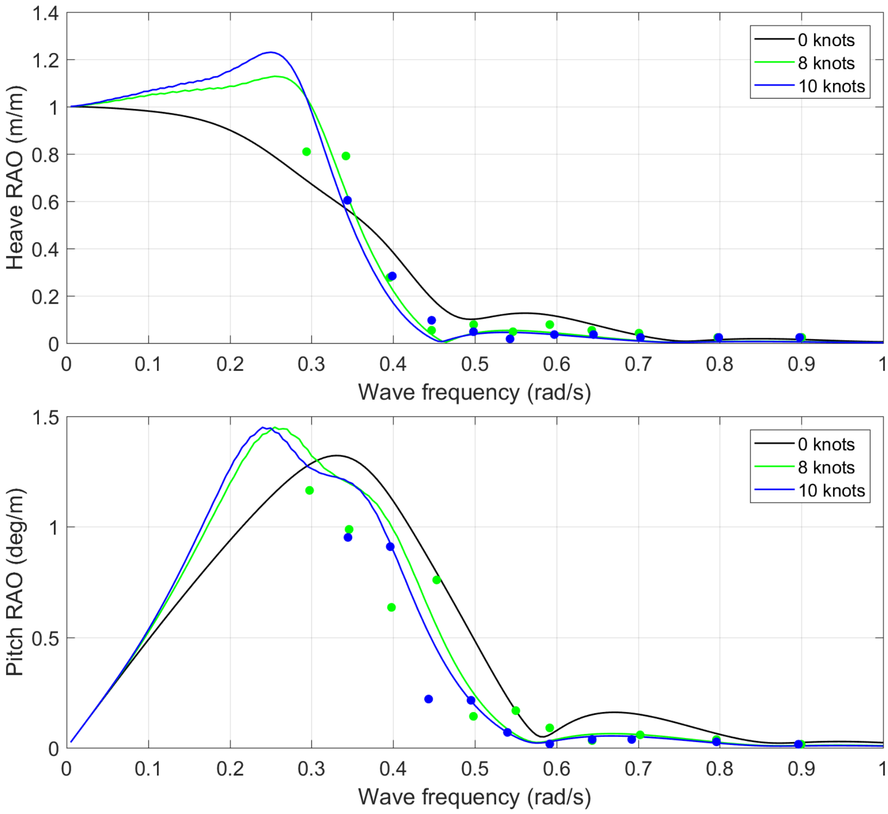


**Figure 1. 190 m bulk carrier in head seas**

We can compare the accuracy of the speed-adjusted panel method presented here, to the strip theory results (OCTOPUS and PDStrip) and Rankine-source panel method (GL Rankine) presented in Gourlay et al. (2015, Fig. 14). All numerical methods give fairly similar results for heave and pitch in head seas.

## 5 RESULTS FOR A 200 M CONTAINER SHIP

We consider the model test case of the Panamax container ship “Ship F” which was tested at 0, 8, 12 knots in head seas at Flanders Hydraulics (Vantorre & Journée 2003). Model test conditions are shown in Table 3.

**Table 3. 200 m container ship model test conditions**

| Parameter | Value | Source |
|---|---|---|
| LOA | 200.00 m | Vantorre & Journée (2003, Table 1) |
| LBP | 190.00 m | " |
| Beam | 32.00 m | " |
| Maximum draft | 11.60 m | " |
| $c_B$ | 0.60 | " |
| Tested draft | 11.6 m | Vantorre & Journée (2003, Table 3) |
| KG | 11.6 m | " |
| Water depth | 13.6 m | " |
| $k_{yy}$ | 25% LBP | " |
| $k_{xx}$ | 36.6% Beam | Gourlay et al. (2015) |

A 4320-panel surface mesh for Ship F, up to the modelled waterline, was developed with OCTOPUS as shown in Gourlay et al. (2015, Fig. 2). This hull mesh was modelled with WAMIT v7.5, using the inputs shown in Table 3. Heave and pitch motions were calculated at 0, 8, 12 knots using the method described in Section 3. Results are shown in Figure 2.

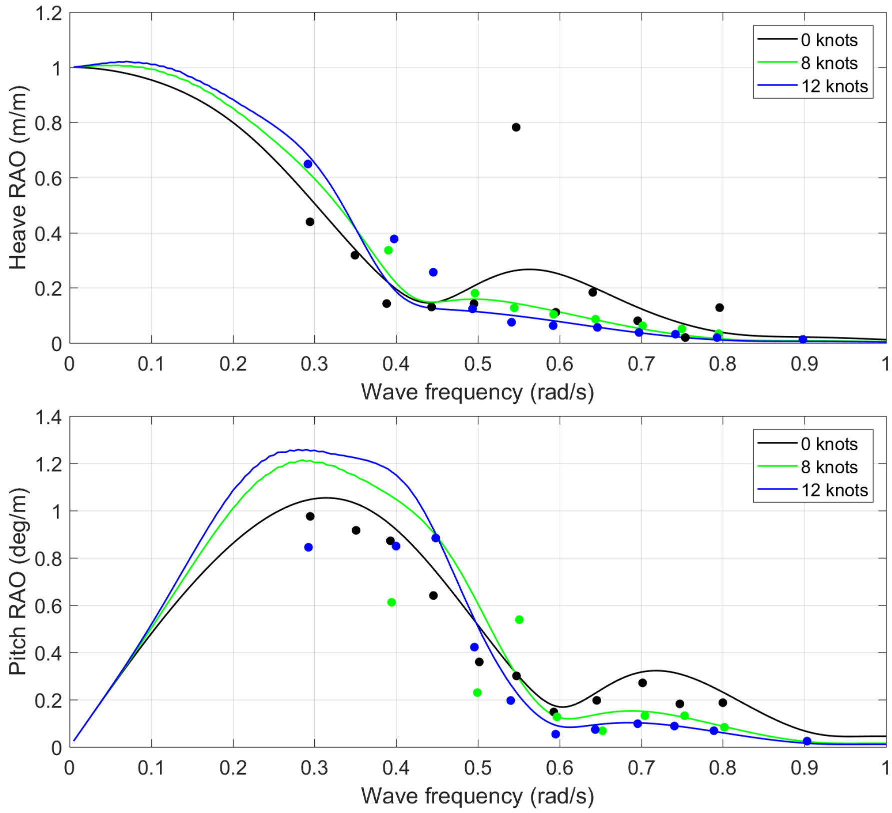


**Figure 2. Comparison of speed-adjusted WAMIT (lines) with model test results (circles), for 200 m container ship in head seas**

As discussed in Gourlay et al. (2015), the model-test zero-speed heave result at 0.55 rad/s is an anomaly, due to the finite width of the tank causing heave resonance at this frequency.

Comparing the accuracy of the speed-adjusted panel method presented here, to the strip theory results (OCTOPUS and PDStrip) and Rankine-source panel method (GL Rankine) presented in Gourlay et al. (2015, Fig. 11), we see that all numerical methods give fairly similar results for heave and pitch in head seas.

## 6 RESULTS FOR A 284 M CONTAINER SHIP

We consider the model test case of the Panamax container ship, which was tested in the Netherlands Ship Model Basin as described in Flokstra (1974), Wahab and Vink (1975) and Zhou et al. (1996). Some of these model test results are compared with numerical methods in Abdelwahab et al. (2023). Model test conditions are shown in Table 4.

**Table 4. 284 m container ship model test conditions**

| Parameter | Value | Source |
|---|---|---|
| LOA | 284.0 m | Zhou et al (1996, Table 1) |
| LBP | 270.0 m | Flokstra (1974, Table 1) |
| Beam | 32.2 m | " |
| Tested draft | 10.85 m | " |
| $c_B$ | 0.60 | " |
| KG | 13.49 m | " |
| GM | 1.15 m | " |
| Water depth | Deep | " |
| $k_{yy}$ | 24.8% LBP | " |
| $k_{xx}$ | 37.5% Beam | " |

Heave, roll and pitch motions were calculated at 0, 10, 24.5 knots using the method described in Section 3. Results are shown in Figure 3 to Figure 5. The horizontal axis is (λ / LBP), and roll and pitch are plotted as a fraction of wave slope, to enable direct comparison with the numerical results in Flokstra (1974) and Abdelwahab et al. (2023).

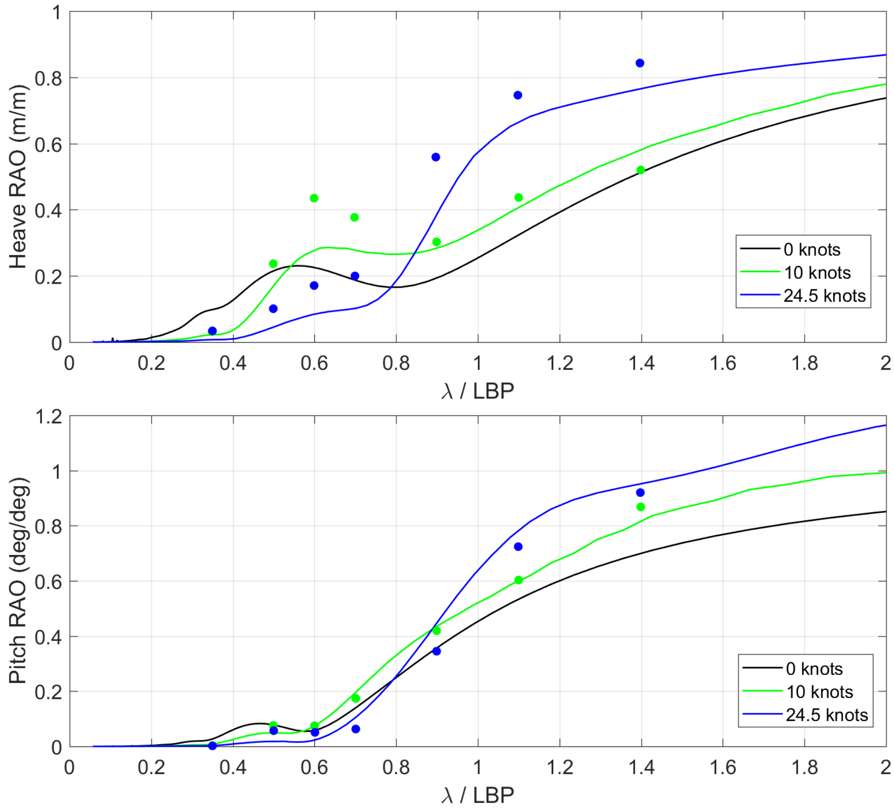


**Figure 3. 284 m container ship in head seas**

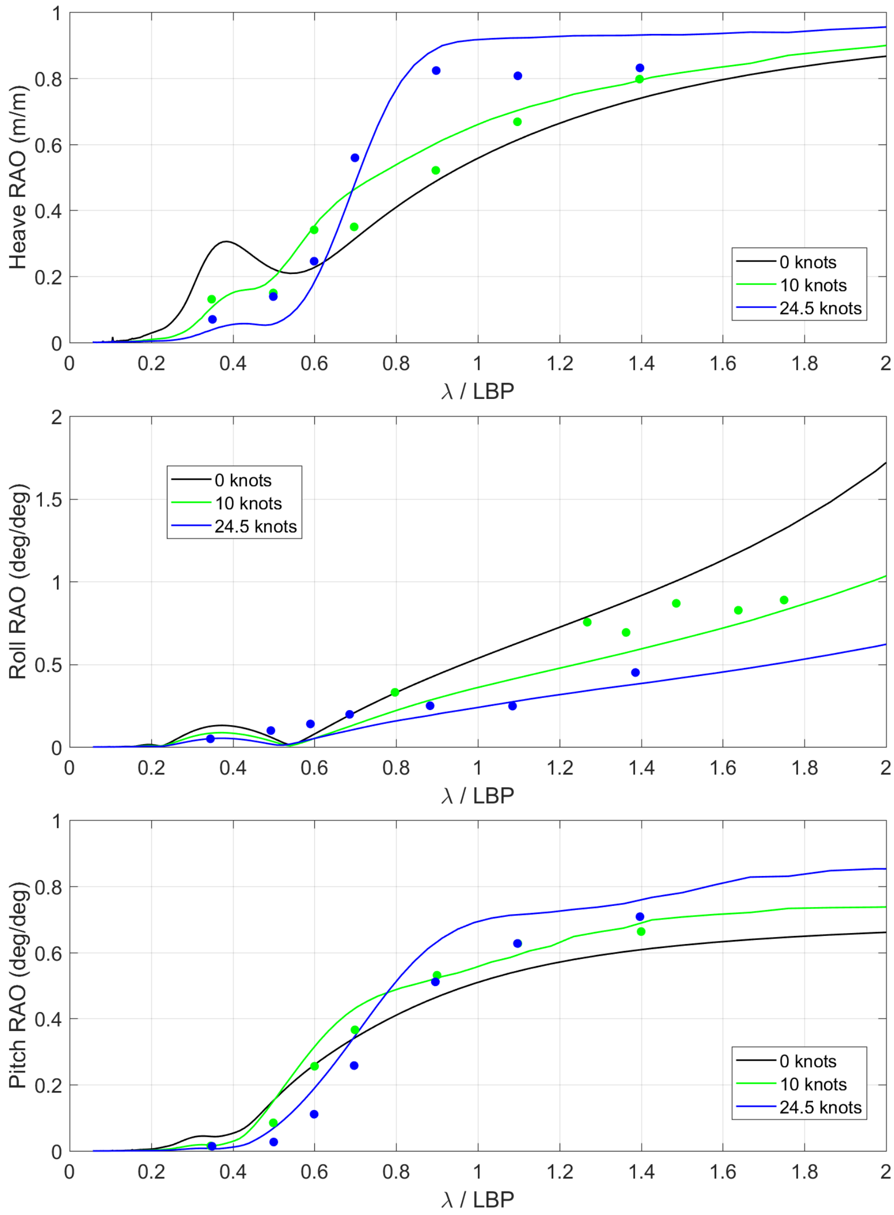


**Figure 4.** **284 m container ship in bow-quartering seas**

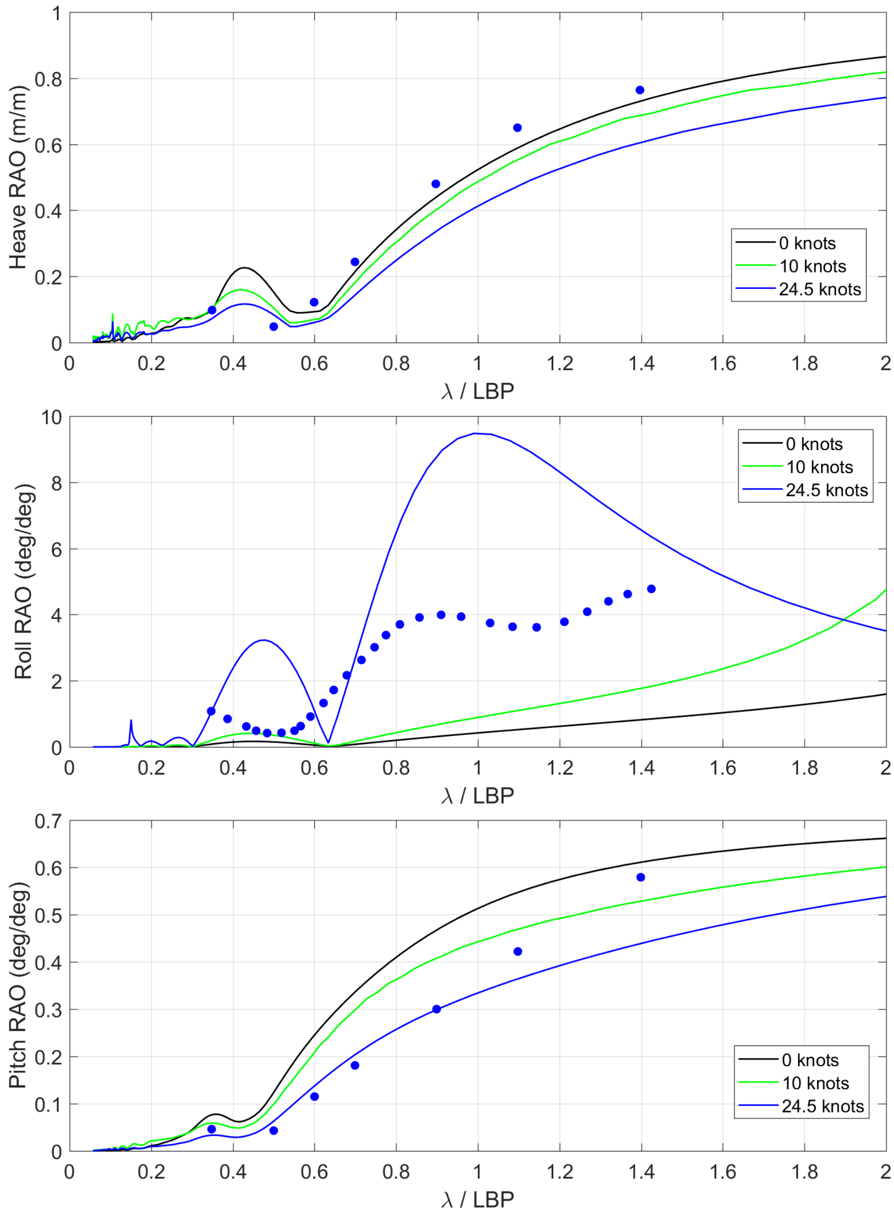


**Figure 5. 284 m container ship in stern-quartering seas**

We can compare the accuracy of the speed-adjusted panel method presented here, to the strip theory results presented in Flokstra (1974). As may be expected, because this Panamax container ship is a slender hull, the speed-adjusted panel method and strip theory method give fairly similar results for heave and pitch at all headings. The speed-adjusted panel method gives more accurate roll results in bow-quartering and stern-quartering seas.

Abdelwahab et al. (2023, Figs. 5,11) give results from the strip theory PDStrip for these same cases; these results are of similar accuracy to the speed-adjusted panel method presented here.

## 7 RESULTS FOR A 300 M CONTAINER SHIP

We consider the model test case of a 6,000-TEU container ship "Ship D" which was tested at Flanders Hydraulics (Vantorre & Journée 2003). We shall focus on the tests done at a full-scale speed of 12 knots, at wave headings of $\beta = 10°$, 180°, as studied in Gourlay et al. (2015). Model test conditions are shown in Table 5.

**Table 5. 300 m container ship model test conditions**

| Parameter | Value | Source |
|---|---|---|
| LOA | 300.00 m | Vantorre & Journée (2003, Table 1) |
| LBP | 291.13 m | " |
| Beam | 40.25 m | " |
| Maximum draft | 15.00 m | " |
| $c_B$ | 0.60 | " |
| Tested draft | 15.00 m | Gourlay et al. (2015) |
| Water depth | 18.0 m | " |
| $k_{yy}$ | 25% LBP | " |
| $k_{xx}$ | 33% Beam | " |
| KG | 15.00 m | Vantorre & Journée (2003, Table 2) |

A 1744-panel surface mesh for Ship D, up to the modelled waterline, was developed with OCTOPUS as described in Gourlay et al. (2015). This hull mesh was modelled with WAMIT v7.5, using the inputs shown in Table 5.

Heave, pitch and roll motions were calculated at 0 knots and 12 knots using the method described in Section 3. No viscous roll damping was included. Results are shown in Figure 6 and Figure 7.

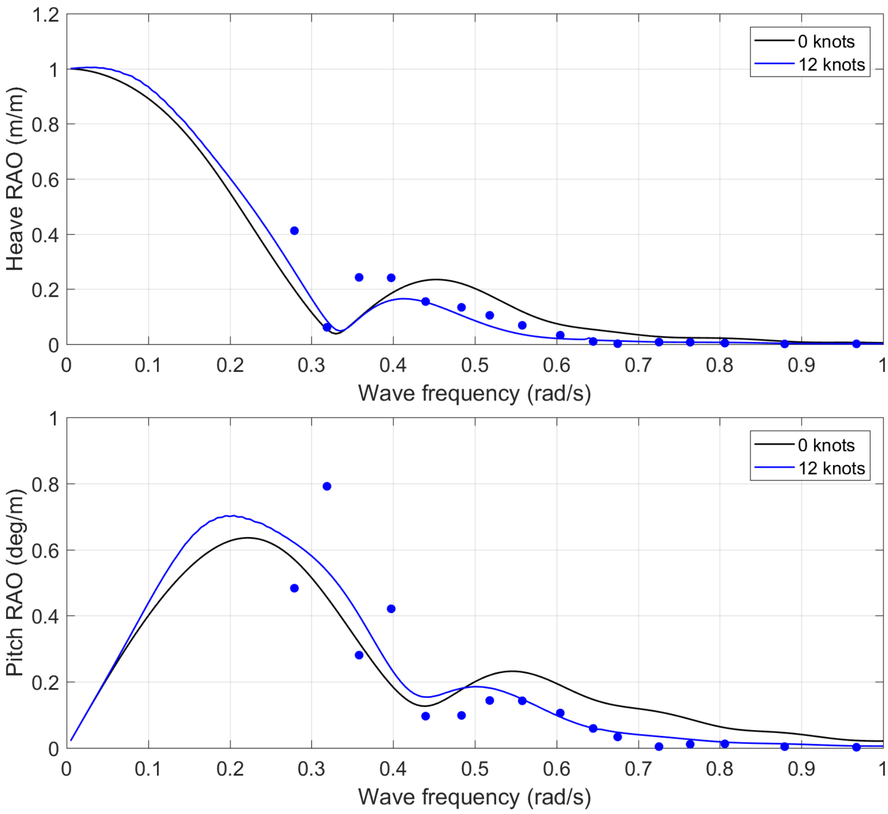


**Figure 6. 300 m container ship in head seas**

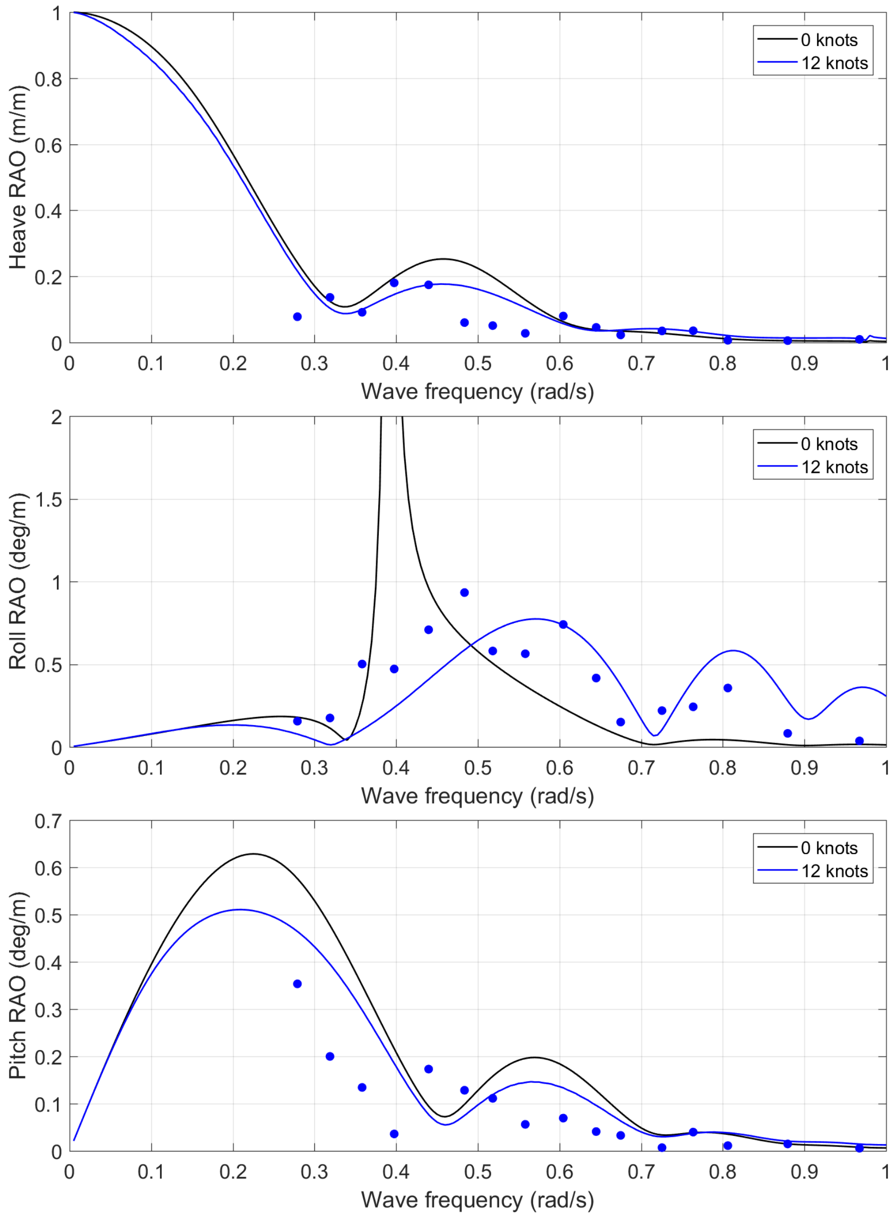


**Figure 7.** **300 m container ship in waves 10° off-stern**

We can compare the accuracy of the speed-adjusted panel method presented here, to the strip theory results (OCTOPUS and PDStrip) and Rankine-source panel method (GL Rankine) presented in Gourlay et al. (2015, Fig. 16). All numerical methods give fairly similar results for this ship.

## 8 RESULTS FOR A 373 M CONTAINER SHIP

Here, we calculate results for the Duisburg Test Case container ship, for test cases CW1-CW5, as described in Van Zwijnsvoorde et al. (2019). The cases that were "blind" in that paper have since been kindly supplied by Ghent University and Flanders Hydraulics. Model test conditions are shown in Table 6.

**Table 6. 373 m container ship model test conditions**

| Parameter | Value | Source |
|---|---|---|
| LOA | 373.0 m | IGES file |
| LBP | 355.0 m | Van Zwijnsvoorde et al. (2019) |
| Beam | 51.00 m | " |
| Maximum draft | 14.50 m | " |
| $c_B$ | 0.66 | " |
| Tested draft | 14.5 m | " |
| KG | 19.8 m | " |
| $k_{yy}$ | 24% LBP | " |
| $k_{xx}$ | 38.9% Beam | " |

Test cases CW1-CW5 are described in Table 7.

**Table 7. 373 m container ship model test cases**

| Test case | Static UKC (% draft) | Speed (full-scale) | Wave height (full-scale) | Wave period (full-scale) |
|---|---|---|---|---|
| CW1 | 100% | 0 knots | 4.86 m | 13.0 s |
| CW2 | 100% | 6 knots | 5.55 m | 13.0 s |
| CW3 | 100% | 16 knots | 5.56 m | 13.0 s |
| CW4 | 20% | 0 knots | 1.98 m | 15.7 s |
| CW5 | 20% | 6 knots | 1.90 m | 15.7 s |

Motion calculations from various ship motion codes for these test cases are described in Gourlay et al. (2019). Table 8 and Table 9 show calculations for these model test cases, using the method described in Section 3.

**Table 8. 373 m container ship vertical motion @AP**

| Test case | $z_a/\zeta$ model test | $z_a/\zeta$ WAMIT | $\varepsilon_a$ model test | $\varepsilon_a$ WAMIT |
|---|---|---|---|---|
| CW1 | 0.327 | 0.270 | 53 | 58 |
| CW2 | 0.338 | 0.310 | 48 | 31 |
| CW3 | 0.274 | 0.254 | 22 | -20 |
| CW4 | 0.270 | 0.348 | 36 | 59 |
| CW5 | 0.276 | 0.340 | 34 | 24 |

**Table 9. 373 m container ship vertical motion @FP**

| Test case | $z_f/\zeta$ model test | $z_f/\zeta$ WAMIT | $\varepsilon_f$ model test | $\varepsilon_f$ WAMIT |
|---|---|---|---|---|
| CW1 | 0.504 | 0.526 | -174 | 179 |
| CW2 | 0.363 | 0.565 | 180 | 151 |
| CW3 | 0.346 | 0.387 | 134 | 106 |
| CW4 | 0.401 | 0.440 | 174 | 158 |
| CW5 | 0.268 | 0.352 | 165 | 136 |

We can compare the accuracy of the speed-adjusted panel method presented here, to the other numerical results in Gourlay et al. (2019). This comparison is shown in Figure 8.

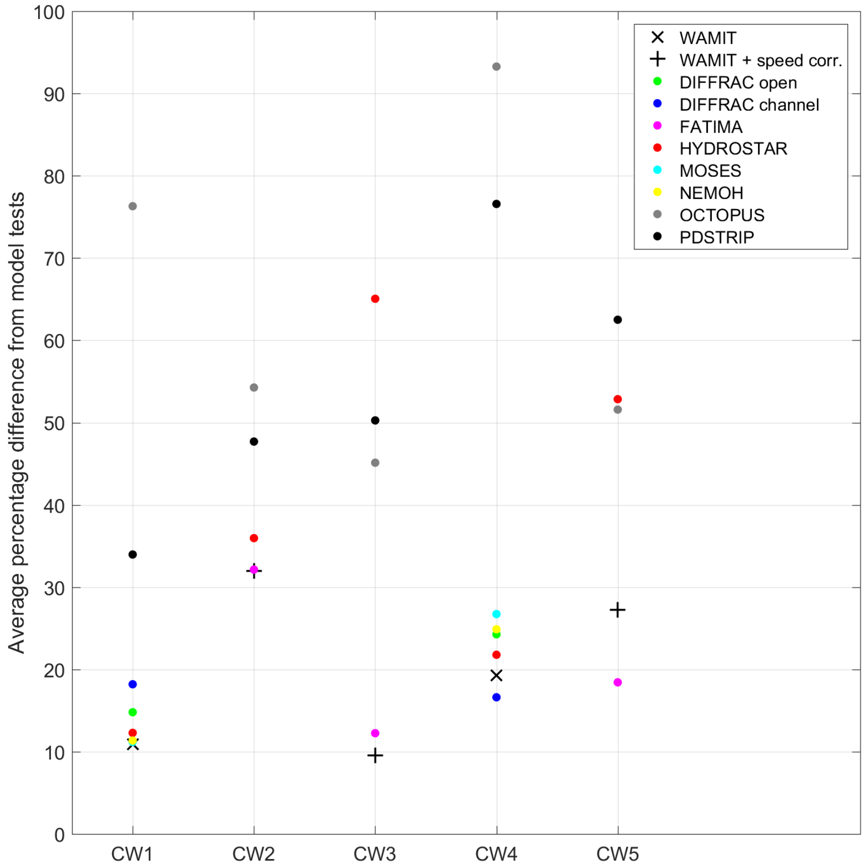


**Figure 8. Percentage difference between numerical methods and model test results, averaged over bow and stern motion amplitudes**

We see that WAMIT is amongst the most accurate methods for the zero-speed cases (CW1 and CW4), while the speed-adjusted WAMIT method is amongst the most accurate methods for the forward-speed cases (CW2, CW3 and CW5).

## 9 CONCLUSIONS

In conclusion, this study presents a computationally efficient method for approximating wave-induced motions of ships at low Froude numbers. By utilizing zero-speed Green functions from WAMIT and adjusting for ship speed, the method demonstrates comparable accuracy to established numerical methods such as strip theory and Rankine-source panel methods. The approach is validated against model test data for various ship types, including bulk carriers and container ships, under different speed and wave conditions. The speed-adjusted panel method can be used to predict ship motions in both shallow and deep water environments. While some error is expected due to the use of zero-speed Green functions, the method's simplicity and efficiency make it a valuable tool for preliminary ship design and performance assessment.

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## 11 AUTHOR BIOGRAPHY

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