The Effect of Combination Mooring Lines on Dynamic Mooring Operability, for Bulk Carriers at Wave-Exposed Berths

Tim Gourlay[1]

[1]Perth Hydro Pty Ltd, Perth, Australia; tim@perthhydro.com

Abstract: Dynamic mooring analysis has been done for a typical Capesize bulk carrier, with standard mooring lines or combination mooring lines (high-modulus polyethylene with soft tail). It was found that combination lines with 22 m nylon tails had similar dynamic mooring behaviour to standard lines. However, the short distance from ship fairlead to shore bollard makes 22 m tails impractical for many berths. If 11 m tails are used, the choice of tail material is a trade-off between loads and motions. 11 m nylon tails showed moderate loads and moderate motions. 11 m PPL/PES tails showed high loads and low motions. 11 m nylon tails gave the best overall operability for the conditions tested, but only a small improvement over standard lines. The decision to use combination lines may come down to their light weight and ease of manual handling, rather than an expected improvement in dynamic mooring behaviour.

Keywords: Mooring lines, dynamic mooring analysis

Nomenclature

ARCSOPT	Association of Resource Companies, Ship Operators, Ports and Terminals
DBCT	Dalrymple Bay Coal Terminal
DoF	Degrees of freedom
GM	Transverse metacentric height above VCG
HMPE	High modulus polyethylene
IRF	Impulse response function
JBC	Japan Bulk Carrier
KG	Centre of gravity height above keel
LBP	Length between perpendiculars
LCG	Longitudinal centre of gravity
LOA	Length overall
MBL	Minimum breaking load
OCIMF	Oil Companies International Marine Forum
PIANC	Permanent International Association of Navigation Congresses
PES	Polyester
PPL	Polypropylene
QTF	Quadratic transfer function
RAO	Response amplitude operator
RMS	Root mean square
VCG	Vertical centre of gravity

Introduction

“Combination mooring lines” consist of a low-stretch HMPE mooring line, combined with a soft tail, often called a “stretcher”. This setup has the following potential advantages over standard mooring lines:

- The HMPE line is typically half the diameter and quarter the weight of a standard PPL/PES line, for the same breaking load, making for easier line handling.
- Having the same length tail on all lines gives them similar stretch characteristics, independent of the distance from fairlead to shore bollard. This means that loads may be more equally shared between lines of different length.

An example standard mooring line is shown in Figure 1, together with a HMPE line of similar strength.

Figure 1 (Top) Standard 64 mm PPL/PES mooring line, MBL 81 tonnes; (Bottom) 33 mm HMPE mooring line, MBL 94 tonnes

Combination mooring lines are commonly used on LNG carriers, as recommended by OCIMF [10, §5.6, 5.8]. Chafe protection is particularly important for the HMPE lines [10, §5.6.7]. The standard tail length is 11 m [10, §5.8.3.1], though 22 m tails are used on some of the larger ships. The tail MBL is recommended by OCIMF to be 125 – 130% of the HMPE line MBL [10, §5.8.2]. Nylon tails are favoured for their high stretch. An example LNG carrier mooring with combination mooring lines is shown in Figure 2.

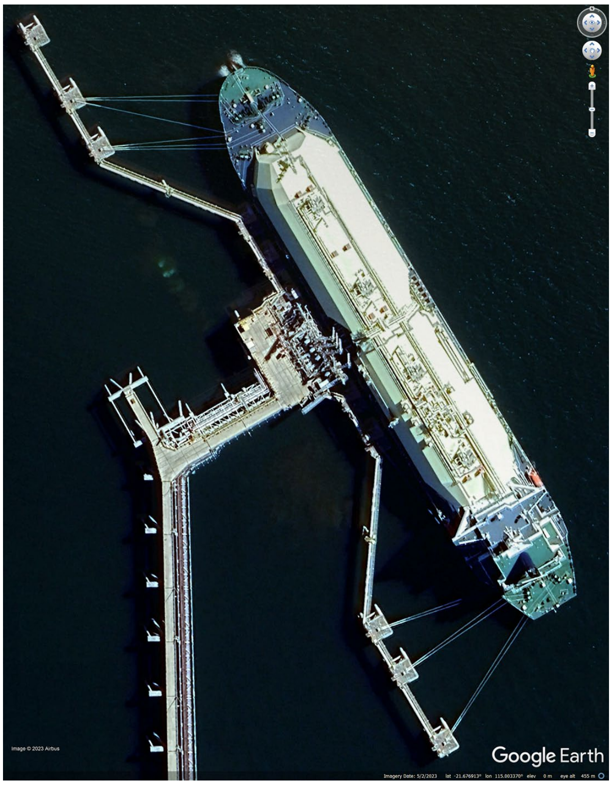

Figure 2 Example LNG carrier mooring at Wheatstone terminal, showing combination mooring lines

Combination mooring lines are commonly used on large container ships [15, §5.2], [18, p1] and on large cruise ships [3].

Figure 3 Cruise ship Sky Princess with combination mooring lines. Photo www.gleistein.com.

An example Capesize bulk carrier with combination mooring lines is shown in Figure 4.

Figure 4 Capesize bulk carrier "Zampa Blue" at Anderson Point Berth 3, Port Hedland. Combination mooring lines consist of 35 mm HMPE main lines (93.5 tonnes MBL) with soft tails.

At the present time, combination mooring lines are used on only a small percentage of bulk carriers. However, this situation may change in future. Dalrymple Bay Coal Terminal is presently phasing-in a requirement for visiting bulk carriers to use HMPE mooring lines with soft tails, due to the exposed nature of the berths [2]. The tails must be 11 m PPL/PES or double-braid nylon, with MBL at least as high as the HMPE line [2]. An example is shown in Figure 5.

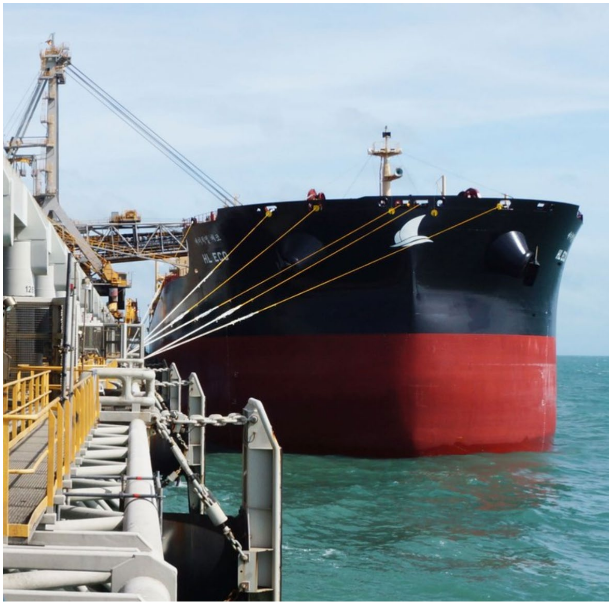

Figure 5 Bulk carrier with combination mooring lines at Dalrymple Bay Coal Terminal. Photo www.dbct.com.au.

The new ARCSOPT mooring guidelines [1] discuss the use of HMPE lines with soft tails. They recommend that:

- Soft tails should always be used with HMPE lines
- Tails should be between 11 m and 22 m in length
- Tails should have MBL approximately 125 – 130% of the HMPE line MBL.

Multi-berth and single-berth shiploaders

For berths with a travelling shiploader serving multiple berths, the mooring lines are constrained not to run across the shiploader rails, so the mooring lines must be run to the front of the berth. Examples are shown in Figure 6 and Figure 7.

Figure 6 Newcastlemax bulk carrier Ever Shine at Nelson Point Berth D, Port Hedland. The multi-berth shiploader means that all mooring lines must be run to the front of the berth, so as not to cross the shiploader rails.

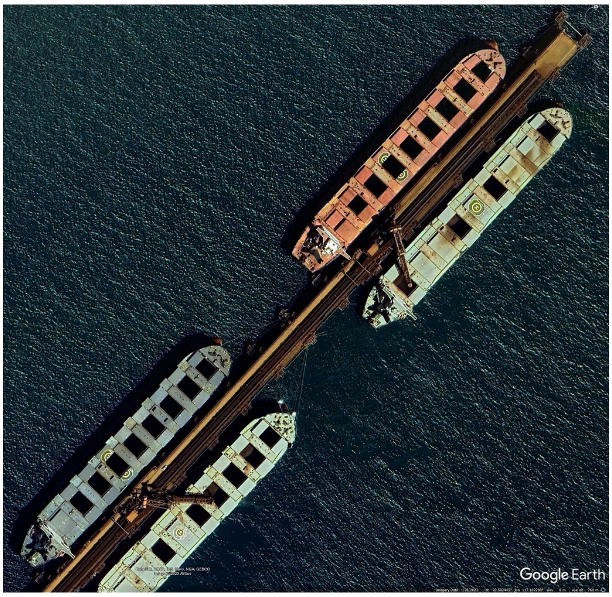

Figure 7 Cape Lambert B berths. All mooring lines are run to the front of the berth, so as not to cross the shiploader rails.

For shiploaders on short rails serving a single berth, long breast lines can be run outside each end of the shiploader rails. An example is shown in Figure 8.

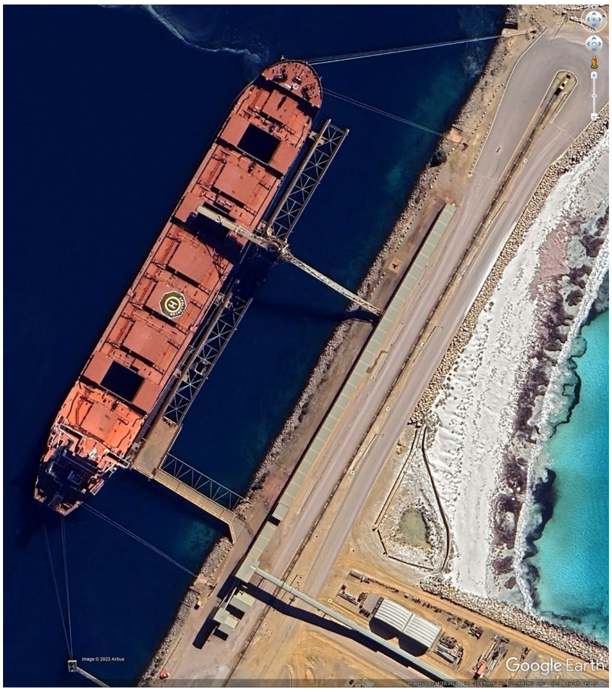

Figure 8 Satellite image of Esperance Berth 3. Long breast lines (and head and stern lines) are run at each end of the shiploader rails.

Test case

For this study, we model a standard Capesize bulk carrier at generic bulk berths with short or long breast lines. The berths are modelled as open trestle berths, in open water of constant depth, to simplify wave loading on the ship. The study undertakes a direct comparison of moored ship wave-induced motions and loads, with standard mooring lines or combination mooring lines.

Modelled hullform

A Newcastlemax bulk carrier is modelled in this study. The Newcastlemax is a mid-range Capesize vessel with maximum dimensions 300 x 50 m. Dimensions and general arrangement for the modelled vessel are taken from the example vessel "China Steel Team" [17]. Dimensions are shown in Table 1.

Table 1 Modelled ship dimensions

LOA	299.50 m
LBP	290.50
Beam	50.00 m
Depth	24.40 m
Design draft	16.00 m
Scantling draft	18.07 m
Scantling deadweight	203,512 t

The hull shape is modelled using the JBC parent hull [8], a standard Capesize bulk carrier. The JBC hull is scaled to the modelled length between perpendiculars, beam and design draft shown in Table 1. A surface mesh for the modelled hull is developed using OCTOPUS software, as shown in Figure 9.

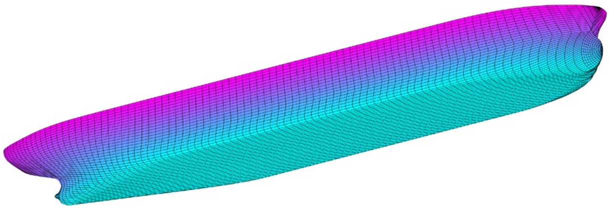

Figure 9 7492-panel surface mesh of JBC bulk carrier, meshed to main deck level

The ship is modelled in fully-loaded condition, as shown in Table 2.

Table 2 Modelled loading condition

Draft aft	18.07 m
Draft forward	18.07 m
DIsplacement	232,200 tonnes
KG	10.07 m
GM	10.80 m

Modelled mooring lines

As described in [14, Table 8-3], Capesize bulk carriers of 150,000 – 230,000 deadweight tonnes, with mixed PPL/PES mooring lines, tend to use line diameter of 64 – 96 mm, with MBL of 70 – 135 tonnes. We choose a standard mooring line in the middle of this range, as described in Table 3.

Table 3 Modelled standard mooring lines

Type	Akwaflex®
Material	70% PPL, 30% PES
Lay	12-strand
Reference	www.jeyco.com.au
Diameter	72 mm
Length	200 m
MBL	102.0 tonnes
Weight	2.84 kg/m
Elongation at break	16%

Line types used for combination mooring lines are described in Table 4 to Table 6. Following ARCSOPT guidelines [1], we model tail lengths of 11 m and 22 m, with tail MBL approximately 125 – 130% of the HMPE line MBL.

Table 4 Modelled HMPE lines

Type	Amsteel Blue®
Material	HMPE
Lay	12-strand
Reference	www.samsonrope.com
Diameter	38 mm
Length	200 m
MBL	104.0 tonnes
Weight	0.86 kg/m
Elongation at break	3%

Table 5 Modelled nylon tails

Material	Nylon
Lay	Double braid
Reference	www.jeyco.com.au
Diameter	72 mm
Length	11 m or 22 m
MBL	125.6 tonnes
Weight	3.49 kg/m
Elongation at break	30%

Table 6 Modelled PPL/PES tails

Type	Akwaflex®
Material	70% PPL, 30% PES
Lay	12-strand
Reference	www.jeyco.com.au
Diameter	80 mm
Length	11 m
MBL	124.0 tonnes
Weight	3.49 kg/m
Elongation at break	16%

Tension vs fractional extension for each rope type are shown in Figure 10.

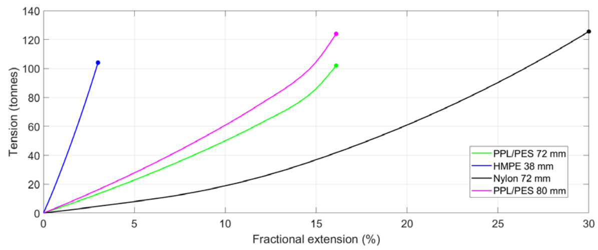

Figure 10 Tension vs fractional extension for rope types considered in this study

It is assumed that all mooring lines are free to move through the fairlead when the line stretches. Allowance is made for line stretch between the ship winch and ship fairlead, and between the ship fairlead and shore bollard.

Figure 11 Moored Capesize bulk carrier with aft breast lines through roller and Panama chock. Dynamic line stretch between winch and fairlead is important for standard mooring lines.

PIANC [14, §8.2.4] recommend using a mooring line pre-tension of 5 – 10% MBL for bulk carriers. We use 5% MBL for the breast lines (which are shortest and most prone to breaking) and 10% MBL for all other lines.

Modelled fenders

Fenders modelled are a single 1.80 m cone fender on each dolphin. Each fender has rated reaction 298 tonnes. Each fender is fitted with a low-friction facing panel with a modelled coefficient of friction of 0.2.

Modelled wave conditions

In order to excite 6-DoF wave-induced motions of the moored ship, we consider a principal wave direction on the port bow quarter. Modelled wave conditions are shown in Table 7.

Table 7 Modelled wave conditions

	Short-period swell case	*Long-period swell case*
Peak wave period	10 seconds	15 seconds
Significant wave height (< 25 s)	2.0 m	1.0 m
Significant long wave height (25 – 200 s)	0.12 m	0.10 m
	Both swell cases	
Peak wave direction	45˚ off port bow	
Wave spectral shape (< 25 s)	Bretschneider standard spectrum	
Peak long wave direction	45˚ off port bow	
Long wave spectral shape	Constant spectral density 25 – 200 s	
Wave spreading	Cosine-squared wave spreading, maximum spread ±45˚	

Dynamic mooring analysis

Dynamic mooring analysis is done using nonlinear time-domain software, following PIANC guidelines [16]. The nonlinear time-domain solver MoorMotions (www.moormotions.com) is used for the modelling. The software and its validation are described in [4,5,12]. The equation of motion is described in Appendix A.

A timestep of 0.1 s is used for all time-domain simulations. Maximum values are averaged over ten 1-hour simulations, to give expected 1-hour maximum values.

First-order wave loads are calculated from the wave load RAOs. Second-order wave loads are calculated from the moored ship's QTFs, using the method described in [9, eq. 4] and [11, p167].

Wave load RAOs and QTFs, and hydrodynamic IRFs, are calculated using WAMIT software [19]. WAMIT settings are shown in Table 8.

Table 8 WAMIT settings for hydrodynamic modelling

WAMIT solver	Direct solver, standard velocity potential
Water depth	20 m, including tide
Degrees of freedom	Coupled 6-DoF
1st - order wave loads	Diffraction potential
2nd - order wave loads	Momentum balance
WAMIT wave headings	0˚ : 15˚ : 360˚ (25 off)
Wave frequencies, for wave loading	0.03 : 0.005 : 1.53 rad/s (301 off)
Wave frequencies, for IRFs	0.0 : 0.003 : 2.1 rad/s, plus infinite frequency (702 off)
Time vector for IRFs	0.1 s timestep, total 300 s
Roll gyradius	35% beam
Pitch gyradius	25% LOA
Yaw gyradius	25% LOA
Viscous damping	Bilge keel roll damping [6] and eddy roll damping [7]

Nonlinear free decay tests were firstly undertaken, to find the natural periods of the mooring system. These are shown in Table 9.

Table 9 Natural motion periods of mooring system with short breast lines (see Figure 12)

	Standard lines	*Combi lines, 11m nylon tails*	*Combi lines, 22m nylon tails*	*Combi lines, 11m PPL /PES tails*
Surge	126 s	94 s	114 s	90 s
Sway	120 s	109 s	123 s	93 s
Heave	23 s	23 s	23 s	23 s
Roll	14 s	14 s	14 s	14 s
Pitch	18 s	18 s	18 s	18 s
Yaw	93 s	74 s	90 s	68 s

Dynamic motion and load operability criteria

Dynamic motion and load operability criteria are shown in Table 10.

Table 10 Dynamic motion and load operability criteria for moored bulk carriers with conveyor belt loading

Parameter	*Limit*	*Source*
Surge amplitude (away from mean position)	2.50 m	[13, Table 4-9]
Sway amplitude (away from fender line) at Hold 1	2.50 m	[13, Table 4-9]
Sway amplitude (away from fender line) at Hold 9	2.50 m	[13, Table 4-9]
Dynamic mooring line load	51.0 tonnes	Based on 50% of minimum MBL [10]
Fender compression	1.30 m	Rated compression 72% x 1.80 m

Berth with short breast lines

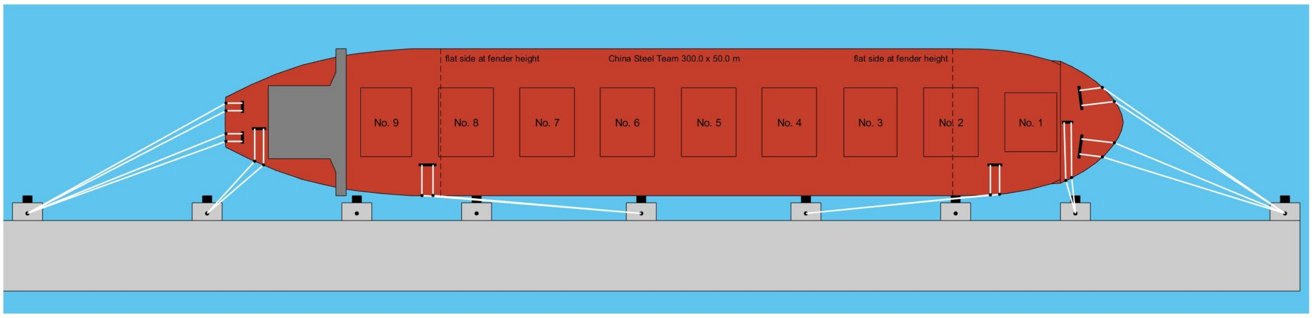

Figure 12 Modelled mooring arrangement with short breast lines

Dynamic mooring analysis results are shown in Table 11 and Table 12, for the short-period swell case shown in Table 7. Highlighted orange values exceed the operability limits in Table 10.

Table 11 RMS motions for berth with short breast lines, in short-period swell case

	Standard lines	*Combi, 11m nylon tails*	*Combi, 22m nylon tails*	*Combi, 11m PPL/ PES tails*
Surge	0.44 m	0.38 m	0.45 m	0.35 m
Sway	0.38 m	0.33 m	0.42 m	0.27 m
Heave	0.04 m	0.04 m	0.04 m	0.04 m
Roll	0.25°	0.25°	0.25°	0.25°
Pitch	0.04°	0.04°	0.04°	0.04°
Yaw	0.20°	0.16°	0.21°	0.14°

Table 12 Averaged 1-hour maximum motions and loads for berth with short breast lines, in short-period swell case

	Stan-dard lines	*Combi, 11m nylon tails*	*Combi, 22m nylon tails*	*Combi, 11 m PPL/ PES tails*
Surge	1.59 m	1.29 m	1.56 m	1.17 m
Sway, Hold 1	2.21 m	2.06 m	2.46 m	1.56 m
Sway, Hold 9	2.64 m	2.02 m	2.68 m	1.60 m
Outer stern	21.5 t	36.6 t	31.6 t	44.2 t
Inner stern	20.5 t	33.6 t	28.7 t	42.2 t
Aft breast	41.6 t	49.8 t	34.0 t	71.1 t
Aft spring	20.6 t	35.5 t	27.0 t	43.0 t
Fwd spring	18.3 t	30.9 t	24.3 t	37.4 t
Fwd breast	37.4 t	52.2 t	31.4 t	66.9 t
Inner head	21.2 t	36.8 t	28.0 t	44.6 t
Outer head	20.2 t	37.6 t	38.4 t	43.7 t
Fender	0.80 m	0.90 m	0.89 m	0.79 m

Dynamic mooring analysis results are shown in Table 13 and Table 14, for the long-period swell case shown in Table 7.

Table 13 RMS motions for berth with short breast lines, in long-period swell case

	Standard lines	*Combi, 11m nylon tails*	*Combi, 22m nylon tails*	*Combi, 11m PPL/ PES tails*
Surge	0.48 m	0.38 m	0.47 m	0.34 m
Sway	0.43 m	0.36 m	0.47 m	0.28 m
Heave	0.09 m	0.09 m	0.09 m	0.09 m
Roll	0.37°	0.37°	0.37°	0.36°
Pitch	0.08°	0.08°	0.08°	0.08°
Yaw	0.22°	0.18°	0.22°	0.17°

Table 14 Averaged 1-hour maximum motions and loads for berth with short breast lines, in long-period swell case

	Stan-dard lines	*Combi, 11m nylon tails*	*Combi, 22m nylon tails*	*Combi, 11 m PPL/ PES tails*
Surge	1.61 m	1.25 m	1.48 m	1.12 m
Sway, Hold 1	2.59 m	1.98 m	2.73 m	1.52 m
Sway, Hold 9	2.76 m	2.32 m	3.06 m	1.88 m
Outer stern	21.9 t	35.1 t	32.1 t	40.0 t
Inner stern	21.0 t	31.8 t	28.5 t	38.2 t
Aft breast	43.3 t	52.1 t	38.5 t	67.6 t
Aft spring	20.9 t	35.8 t	27.8 t	42.2 t
Fwd spring	19.2 t	28.7 t	23.2 t	36.1 t
Fwd breast	43.1 t	47.4 t	35.0 t	63.3 t
Inner head	22.3 t	36.6 t	30.6 t	43.3 t
Outer head	21.4 t	36.4 t	32.3 t	41.7 t
Fender	0.93 m	0.87 m	0.88 m	0.88 m

The results for short breast lines show that:

- Combination lines with 22 m nylon tails have similar motions and loads to standard lines
- Combination lines with 11 m nylon tails have lower motions, higher loads and similar operability to standard lines
- Combination lines with 11 m PPL/PES tails have low motions but high line loads.

Berth with long breast lines

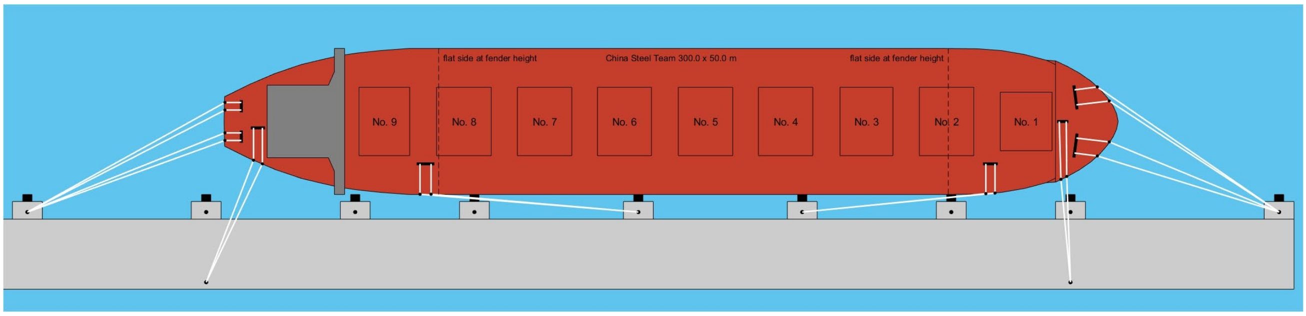

Figure 13 Modelled mooring arrangement with long breast lines

Dynamic mooring analysis results are shown in Table 15 and Table 16, for the short-period swell case shown in Table 7. Highlighted orange values exceed the operability limits in Table 10.

Table 15 RMS motions for berth with long breast lines, in short-period swell case

	Standard lines	*Combi, 11m nylon tails*	*Combi, 22m nylon tails*	*Combi, 11m PPL/ PES tails*
Surge	0.42 m	0.34 m	0.42 m	0.31 m
Sway	0.43 m	0.32 m	0.41 m	0.27 m
Heave	0.04 m	0.04 m	0.04 m	0.04 m
Roll	0.25°	0.25°	0.25°	0.25°
Pitch	0.04°	0.04°	0.04°	0.04°
Yaw	0.22°	0.16°	0.22°	0.13°

Table 16 Averaged 1-hour maximum motions and loads for berth with long breast lines, in short-period swell case

	Stan-dard lines	*Combi, 11m nylon tails*	*Combi, 22m nylon tails*	*Combi, 11 m PPL/ PES tails*
Surge	1.48 m	1.15 m	1.40 m	1.13 m
Sway, Hold 1	2.94 m	2.03 m	2.63 m	1.73 m
Sway, Hold 9	2.43 m	1.77 m	2.54 m	1.49 m
Outer stern	21.4 t	36.3 t	29.7 t	45.2 t
Inner stern	20.8 t	33.9 t	26.5 t	42.9 t
Aft breast	28.3 t	44.8 t	36.3 t	60.1 t
Aft spring	18.1 t	28.9 t	23.2 t	40.0 t
Fwd spring	20.3 t	29.8 t	24.1 t	36.3 t
Fwd breast	32.1 t	47.4 t	35.3 t	63.4 t
Inner head	19.9 t	33.1 t	26.3 t	43.0 t
Outer head	20.5 t	35.6 t	28.0 t	43.8 t
Fender	0.92 m	0.84 m	0.96 m	0.78 m

Dynamic mooring analysis results are shown in Table 17 and Table 18, for the long-period swell case shown in Table 7.

Table 17 RMS motions for berth with long breast lines, in long-period swell case

	Standard lines	*Combi, 11m nylon tails*	*Combi, 22m nylon tails*	*Combi, 11m PPL/ PES tails*
Surge	0.41 m	0.32 m	0.42 m	0.27 m
Sway	0.45 m	0.34 m	0.45 m	0.27 m
Heave	0.09 m	0.09 m	0.09 m	0.09 m
Roll	0.37°	0.37°	0.37°	0.36°
Pitch	0.08°	0.08°	0.08°	0.08°
Yaw	0.25°	0.18°	0.23°	0.15°

Table 18 Averaged 1-hour maximum motions and loads for berth with long breast lines, in long-period swell case

	Stan-dard lines	*Combi, 11m nylon tails*	*Combi, 22m nylon tails*	*Combi, 11 m PPL/ PES tails*
Surge	1.38 m	1.11 m	1.32 m	0.93 m
Sway, Hold 1	3.01 m	2.17 m	2.77 m	1.63 m
Sway, Hold 9	2.78 m	2.06 m	2.74 m	1.60 m
Outer stern	22.1 t	36.2 t	30.8 t	41.8 t
Inner stern	21.0 t	32.3 t	27.5 t	39.2 t
Aft breast	31.7 t	51.4 t	39.1 t	61.7 t
Aft spring	18.1 t	29.7 t	24.0 t	34.9 t
Fwd spring	19.0 t	27.5 t	23.4 t	32.0 t
Fwd breast	32.8 t	52.0 t	36.4 t	60.0 t
Inner head	21.2 t	33.5 t	28.3 t	39.8 t
Outer head	22.1 t	35.8 t	31.9 t	42.3 t
Fender	0.89 m	0.87 m	0.96 m	0.84 m

The results for long breast lines show that:

- Combination lines with 22 m nylon tails show smaller motions than standard lines
- Combination lines with 11 m nylon tails show the best mooring behaviour of the different options
- Combination mooring lines with 11 m PPL/PES tails show small motions but high loads.

Discussion

22 m tails may not work for berths with short breast lines. From OCIMF [10, §5.8.3.1], "*Tails should not be so long that they come into contact with fairleads during operation*". This fact favours the use of 11 m tails for berths with short breast lines (see Figure 5).

If 11 m tails are used, the choice of tail material is a trade-off between loads and motions. Nylon gives moderate loads and moderate motions. PPL/PES gives moderate-to-high loads and low motions. 11 m nylon tails give the best overall operability for the conditions tested.

References

[1] ARCSOPT (2023) Guidelines on safe mooring practices for non-bulk liquid vessels. Technical Guideline 04-23.

[2] DBCT (2023) Dalrymple Bay Coal Terminal – mooring line & vetting requirements – update two, 3rd May 2023.

[3] Gleistein (2023) Mooring solutions – marine industries. Product catalogue.

[4] Gourlay, T.P., Paynter, D.W., Smith, B.J. (2023) Dynamic mooring analysis of 6-buoy spead-moored ships at Cape Cuvier. Proceedings, Coasts and Ports 2023, Sunshine Coast.

[5] Gourlay, T.P. (2019) A coupled ship and harbour model for dynamic mooring analysis in Geraldton Harbour. Proceedings, Coasts and Ports 2019, Hobart.

[6] Himeno, Y. (1981) Prediction of ship roll damping – state of the art. Dept. Naval Architecture and Marine Engineering, University of Michigan, Report No. 239.

[7] Ikeda, Y., Himeno, Y., Tanaka, N. (1978) On eddy making of roll damping force on naked hull. Osaka Prefecture University, Department of Naval Architecture, Report 00403.

[8] National Maritime Research Institute (2015) Tokyo 2015: A Workshop on CFD in Ship Hydrodynamics, http://www.t2015.nmri.go.jp

[9] Newman, J.N. (1974) Second-order, slowly-varying forces on vessels in irregular waves, Marine Vehicles, pp. 182-186.

[10] OCIMF (2018) Mooring Equipment Guidelines, 4th Edition (MEG4).

[11] Orcina (2012) OrcaFlex Manual, Version 9.6a, Orcina Ltd.

[12] Perth Hydro (2019) Comparison of WAMIT and MoorMotions with model tests for a tanker moored at an open berth. Perth Hydro Research Report R2019-09.

[13] PIANC (2023) Criteria for acceptable movement of ships at berths. Report of Working Group 212.

[14] PIANC (2019) Design principles for dry bulk marine terminals. Report of Working Group 184.

[15] PIANC (2012) Criteria for the (un)loading of container vessels. Report of Working Group 115.

[16] PIANC (1995) Criteria for movements of moored ships in harbours. Report of Working Group 24.

[17] Royal Institution of Naval Architects (2007) Significant Ships of 2007, RINA Publications.

[18] Van Zwijnsvoorde, T. (2022) Modelling the behaviour of a moored ship in sheltered waters. PhD thesis, University of Ghent.

[19] WAMIT (2020) WAMIT User Manual, Version 7.4. WAMIT Inc.

Appendix A: Equation of motion for dynamic mooring analysis

MoorMotions uses 6-DoF motion coupling for a moored ship. The equation of motion, as used here, is

$$\sum_{j=1}^{6}[M_{ij}+A_{ij}(\infty)]\ddot{x}_j = X_i^{(1)}+X_i^{(2)}+F_i^{(\text{lines})}+F_i^{(\text{fenders})} + B_i^{(\text{viscous})}-\sum_{j=1}^{6}C_{ij}x_j - \int_0^{\infty}\sum_{j=1}^{6}L_{ij}(\tau)\ddot{x}_j(t-\tau)d\tau$$

The ship origin is the ship centreline, waterline and LCG. The degrees of freedom are shown in Table 19.

Table 19 Ship motion degrees of freedom

x_1	surge (positive forward)
x_2	sway (positive to port)
x_3	heave (positive up)
x_4	roll (positive to starboard)
x_5	pitch (positive bow-down)
x_6	yaw (positive bow-to-port)

Other symbols are defined as follows:

M_{ij} = Mass matrix

$A_{ij}(\infty)$ = Added mass matrix at infinite frequency

$X_i^{(1)}$ = First-order wave load

$X_i^{(2)}$ = Second-order wave load

$F_i^{(\text{lines})}$ = Dynamic mooring line loads on ship, at each instant in time

$F_i^{(\text{fenders})}$ = Fender loads on ship, due to reaction and frictional forces at each instant in time

$B_i^{(\text{viscous})}$ = Viscous damping forces (e.g. roll damping)

C_{ij} = Hydrostatic restoring coefficients

$L_{ij}(\tau)$ = Acceleration-based impulse response functions.