A Coupled Ship and Harbour Model for Dynamic Mooring Analysis in Geraldton Harbour

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Abstract
In this article, we develop a coupled ship-and-harbour model for moored ship motions and loads in Geraldton Harbour. WAMIT software is used to model the ship and harbour as a two-body system. The resulting first-order and second-order wave loads, as well as Impulse Response Functions, are fed into the nonlinear time-domain solver MoorMotions. Timeseries of 6-DoF ship motions, mooring line loads and fender loads are then output. The method is tested, without a ship present, against long wave measurements in Geraldton harbour, showing good agreement. The method is then tested against GNSS measurements of 6-DoF moored ship motions for a Panamax bulk carrier at Geraldton Berth 5, also showing good agreement.

Keywords: ship, mooring, long waves, harbour.

Nomenclature
CoG Centre of gravity
DoF Degrees of freedom
FD Frequency-domain
GMd Roll metacentric height above ship CoG
GNSS Global navigation satellite system
IRF Impulse response function
LBP Length between perpendiculars
LCG Longitudinal centre of gravity
LOA Length overall
MBL Minimum breaking load
MWPA Mid West Ports Authority
PE Polyester
PP Polypropylene
RAO Response amplitude operator
TD Time-domain
UHDPE Ultra-high-density polyethylene
WAMIT Radiation/diffraction panel code

1. Introduction
Geraldton is situated on the mid-west coast of Australia and is exposed to large, long-period Indian Ocean swells from the SW to WSW. The harbour is protected by a headland and reefs, however some swell and long wave energy propagate in through the harbour entrance. A satellite view of the harbour is shown in Figure 1.

Geraldton provides an illustration of the “harbour paradox” which was found by Miles and Munk [8] when studying harbour shapes including the one shown in Figure 2.

Figure 1    Satellite view of Geraldton Harbour. Channel comes north out of the harbour, then curves to the west.

Figure 2   Plan view of idealized harbour used by Miles and Munk [8] to study the “harbour paradox”. As the ratio a/b decreases, long wave energy in the harbour increases.

The harbour paradox states that as the harbour entrance is made smaller, less wave energy can enter the harbour, but less wave energy can also escape from the harbour. The result is that there is less swell energy within the harbour, but more long wave energy.

Principal long wave resonant periods of Geraldton harbour are 128 s and 64 s for E-W waves, and 92 s and 46 s for N-S waves [19]. Moored ship natural surge, sway and yaw periods also fall within this range, leading to the possibility of “double-resonance” when the ship natural motion period is the same as a harbour resonant period.

Much work has been done to understand and predict long waves in Geraldton Harbour and their effect on ship motions, see e.g. [7]. In this paper, we build on these studies and focus on developing a coupled ship-and-harbour model.
2. **Objective**

The objective of this research is to be able to predict maximum ship motions, mooring line loads and fender loads, at any berth within Geraldton Harbour, given the following inputs:

- Ship general arrangement and stability data
- Mooring line and fender details
- Measured, hindcast or forecast swell height and long wave height at a single location within the harbour, between Berth 3 and 4

3. **Test case: Sea Diamond at Berth 5**

On 1st and 2nd October 2015, 6-DoF ship motion measurements were made on the Panamax bulk carrier "Sea Diamond" loading iron ore at Geraldton berth 5 (Figure 3 and Figure 4). These measurements have previously been described in [2,4,5].

When post-processed with GNSS base station data from the pilot jetty, this setup yielded ship motions accurate to 10-12 mm RMS [4].

Ship motion data is taken from 0840-0940 on 2nd October 2015. Mooring lines were let go at 0940 for the ship to depart the berth. Ship data at the time of the measurements is shown in Table 1.

<table>
<thead>
<tr>
<th>LOA</th>
<th>225.0 m</th>
</tr>
</thead>
<tbody>
<tr>
<td>LBP</td>
<td>217.0 m</td>
</tr>
<tr>
<td>Beam</td>
<td>32.26 m</td>
</tr>
<tr>
<td>Draft forward</td>
<td>8.91 m</td>
</tr>
<tr>
<td>Draft aft</td>
<td>10.26 m</td>
</tr>
<tr>
<td>Displacement</td>
<td>57,430 t</td>
</tr>
<tr>
<td>LCG</td>
<td>4.22 m aft of midships</td>
</tr>
<tr>
<td>GMs</td>
<td>5.93 m</td>
</tr>
</tbody>
</table>

The mooring arrangement is shown in Figure 6.

A photo of the stern lines and aft breast lines is shown in Figure 7.
Mooring lines were of two different types:
- 65 mm polypropylene (PP) danline, MBL 57 tonnes, 21% elongation at MBL
- 60 mm mixed polyester (PE) and Nika steel, MBL 63 tonnes, 15% elongation at MBL

Numbers and types of mooring lines are shown in Table 2.

Table 2  Mooring line types, provided by crew of Sea Diamond.

<table>
<thead>
<tr>
<th>Line number</th>
<th>Line</th>
<th>Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>1, 2, 3</td>
<td>Stern lines</td>
<td>65 mm PP</td>
</tr>
<tr>
<td>4, 5</td>
<td>Aft breast lines</td>
<td>60 mm PE</td>
</tr>
<tr>
<td>6, 7</td>
<td>Aft spring lines</td>
<td>60 mm PE</td>
</tr>
<tr>
<td>8, 9</td>
<td>Fwd spring lines</td>
<td>60 mm PE</td>
</tr>
<tr>
<td>10, 11</td>
<td>Fwd breast lines</td>
<td>60 mm PE</td>
</tr>
<tr>
<td>12, 13, 14</td>
<td>Head lines</td>
<td>65 mm PP</td>
</tr>
</tbody>
</table>

Fenders on Berth 5 are Trelleborg SCN1200E1.1 fenders with UHDPE low-friction facing. The fender reaction-compression curve is shown in Figure 8.

Figure 8  Reaction v. Compression curve for Trelleborg SCN1200E1.1 fenders [15], as used on Berth 5.

Measured wave data at the time of the ship motion measurements is shown in Table 3.

Table 3  Measured wave data at Berth 3/4 on 2nd Oct 2015, as supplied by Tremarfon Pty Ltd. Data used in this report is highlighted.

<table>
<thead>
<tr>
<th>Hour ending</th>
<th>0900</th>
<th>1000</th>
<th>1100</th>
</tr>
</thead>
<tbody>
<tr>
<td>Significant swell height (8-25s)</td>
<td>0.13 m</td>
<td>0.14 m</td>
<td>0.17 m</td>
</tr>
<tr>
<td>Significant long wave height (25-120s)</td>
<td>0.09 m</td>
<td>0.13 m</td>
<td>0.12 m</td>
</tr>
</tbody>
</table>

4. Coupled ship and harbour model

Standard mooring analyses for open berths use open-water wave spectra to calculate wave loads on the ship. Van Oortmerssen [17] developed a method to model a ship next to a fully-reflective straight quay wall. A similar method was applied to a ship in Geraldton Harbour by van der Molen et al. [16].

Here we attempt to model the ship in the complete harbour planview geometry. The harbour is modelled as a vertical dipole sheet around the harbour walls. The ship is modelled using a Japan Bulk Carrier standard hull (NMRI 2015) scaled to the correct dimensions and meshed with source panels. The resulting two-body surface mesh is shown in Figure 9 and Figure 10.

Figure 9  Dipole sheet, shown as red line, around the harbour walls. The shallow regions in the tug harbour and near the eastern breakwater are excluded.

Figure 10  Perspective view of harbour mesh (1140 dipole panels) and ship surface mesh (2012 source panels) for MV Sea Diamond at Berth 5 Geraldton.

The method is simplified to assume a constant water depth of 13.0 m (including tide) within the harbour.

5. WAMIT calculations for coupled system

The radiation/diffraction code WAMIT (www.wamit.com) was used to model the coupled system. The use of WAMIT for multi-body problems is described in [18].

External roll stiffness was included to achieve the correct GMx. External roll damping was included to account for viscous bilge keel damping [6], assuming 0.4 m width bilge keels from 40-75% LBP. Ship roll gyradius was taken to be 35% of beam, and pitch gyradius 25% of LOA.

For the WAMIT calculations, 500 evenly-spaced frequencies were used from 0.005 rad/s to 2.5 rad/s. Calculations were also done at zero frequency and infinite frequency, in order to calculate the IRFs.
6. Far-field wave spectrum

The coupled ship and harbour model concentrates on flow within the harbour. Outside the harbour, the water is modelled as open ocean, with depth equal to the depth inside the harbour. Therefore, the method does not model wave propagation across the reefs towards the harbour entrance. Methods for modelling wave propagation outside the harbour are described in [7].

The far-field wave spectrum is calculated to match the measured Berth 3/4 wave conditions shown in Table 3. It is simplified to take a constant value across the range of swell periods (8-25s) and a different constant value across the range of long wave periods (25-200s). Wave direction outside the harbour is assumed to be from the north. Testing with wave directions 15° either side of this made little difference to the results.

A WAMIT wave diffraction analysis is done without a ship present, to relate wave conditions at Berth 3/4 to far-field wave spectra. The far-field wave spectrum corresponding to the wave conditions shown in Table 3 is shown in Table 4. Using the far-field wave spectrum shown in Table 4, the WAMIT-calculated wave spectrum at Berth 3/4 is shown in Figure 11, together with measured results from the port’s Digiquartz pressure sensor.

We see that the wave spectral peaks predicted using WAMIT diffraction analysis are generally close to the measured peaks. An exception is the predicted peak at 140 s, which occurs at 160 s in the measured data. Modelled data using the full-bathymetry code Funwave [3, Fig. 3] confirms this peak at 160 s.

Table 4 Calculated far-field wave spectrum corresponding to wave conditions shown in Table 3.

<table>
<thead>
<tr>
<th></th>
<th>Significant swell height (8-25s)</th>
<th>Significant long wave height (25-200s)</th>
<th>Wave spectral density (8-25s)</th>
<th>Wave spectral density (25-200s)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.072 m</td>
<td>0.077 m</td>
<td>0.00061 m²/(rad/s)</td>
<td>0.00165 m²/(rad/s)</td>
</tr>
</tbody>
</table>

7. Time-domain ship motions and loads

The time-domain solver MoorMotions (www.moormotions.com) was used to calculate motions and loads in the time domain. MoorMotions uses the fourth-order Runge-Kutta time-stepping method [13, p710] to solve the equation of motion [17, eq. 4.23]:

\[
\sum_{j=1}^{6} \left[ M_{ij} + A_{ij}(\infty) \right] \dddot{x}_j = X_1^{(1)} + X_2^{(2)} + F_{i \text{ (lines)}} + F_{i \text{ (fenders)}} - \sum_{j=1}^{6} C_{ij} x_j - \int_{0}^{\infty} \sum_{j=1}^{6} L_{ij}(\tau) \dddot{x}_j(t-\tau) d\tau
\]

The symbols are defined as follows:

- \( x_j \) = motion in each degree of freedom, \( j = 1, ..., 6 \)
- \( M_{ij} \) = mass matrix [9, p307]
- \( A_{ij}(\infty) \) = added mass at infinite frequency
- \( X_1^{(1)} \) = first-order wave load
- \( X_2^{(2)} \) = second-order wave load
- \( F_{i \text{ (lines)}} \) = net force produced by mooring line tension at each instant in time
- \( F_{i \text{ (fenders)}} \) = net force produced by fenders at each instant in time
- \( C_{ij} \) = linear restoring coefficients
- \( L_{ij}(\tau) \) = potential added mass and damping IRF

The coordinate system used is:

- \( x_1 \) = “surge” (fore-aft CoG motion, positive forward)
- \( x_2 \) = “sway” (transverse CoG motion, positive port)
- \( x_3 \) = “heave” (vertical CoG motion, positive up)
- \( x_4 \) = “roll” (angle, positive to starboard)
- \( x_5 \) = “pitch” (angle, positive bow-down)
- \( x_6 \) = “yaw” (angle, positive bow-to-port).
First-order wave loads are calculated from the WAMIT coupled ship-and-harbour wave load RAOs on the ship, together with the spectral wave amplitude at each frequency, as described in [17, eq. 4.26]. Care is taken to ensure independent random phasing of all the input wave frequencies.

Second-order wave loads considered here are the “difference-frequency” and “mean-drift” wave loads. Difference-frequency wave loads use the Newman approximation [10, eq. 9], with the arithmetic mean of the diagonal elements. “Sum-frequency” wave loads are neglected. Calculation of second-order ship wave loads was done using the surrounding control surface shown in Figure 12.

![Image](image1.png)

Figure 12  Control surface surrounding ship, as used for second-order wave load calculations in WAMIT

Additional MoorMotions settings are described in Table 5.

| Mooring line energy release ratio | 65% [1, p32] for PP lines |
| Fender friction coefficient | 0.2 (steel to UHDPE) |
| Fender energy release ratio | 75% [14, p358] |
| Time step | 0.1 seconds |
| Simulation time | 3600 seconds |

### 8. Impulse Response Functions

Time-domain IRFs are calculated from frequency-domain added mass and damping using the WAMIT f2t utility [18]. Added mass calculations for Sea Diamond in open water and at Berth 5 are shown in Figure 13.

As can be seen from Figure 13, added mass coefficients from the coupled ship-and-harbour model are erratic, especially near the harbour resonance periods illustrated in Figure 11. At low frequencies, the coupled added mass is generally slightly larger than the open-water value, as shown in Table 6. This is primarily due to the effect of the nearby rock wall.

When attempting to solve the coupled equations of motion using IRFs from the coupled ship-and-harbour model, it was found that the erratic added mass translated into erratic IRFs, and hence instability in the time-domain simulations. Therefore, here we take the approach of using open-water IRFs in the time-domain calculations. These are shown in Figure 14.

| Table 6  Ratio between added mass of Sea Diamond at Berth 5, to added mass in open water, at low-frequency limit |
| A_{11} | 1.43 |
| A_{22} | 1.18 |
| A_{33} | Erratic |
| A_{44} | 1.04 |
| A_{55} | Erratic |
| A_{66} | 1.09 |

![Image](image2.png)

Figure 13  Calculated added mass diagonal coefficients for Sea Diamond, as calculated in open water or at Berth 5. Water depth used is 13.0 m in each case.
9. Moored ship natural periods

Free decay calculations show that the moored ship has natural motion periods as shown in Table 7.

Table 7  Moored ship natural periods. Horizontal motion periods depend on motion amplitude.

<table>
<thead>
<tr>
<th>Mooring line pre-tension</th>
<th>Surge (s)</th>
<th>Sway (s)</th>
<th>Heave (s)</th>
<th>Roll (s)</th>
<th>Pitch (s)</th>
<th>Yaw (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5 tonnes</td>
<td>115 – 130</td>
<td>80 – 90</td>
<td>13</td>
<td>11</td>
<td>12</td>
<td>60 – 65</td>
</tr>
<tr>
<td>5.0 tonnes</td>
<td>85 – 95</td>
<td>60 – 70</td>
<td>13</td>
<td>11</td>
<td>12</td>
<td>55 – 60</td>
</tr>
</tbody>
</table>

10. Frequency-domain ship motions

Moored ship vertical motions (heave, roll and pitch) may be calculated in the frequency domain [12], by neglecting the effects of mooring lines, fenders and second-order wave loads.

Here, we combine ship heave, roll and pitch RAOs, from the WAMIT coupled ship-and-harbour model, with the far-field wave spectra given in Table 4, to determine the heave, roll and pitch response spectra in the frequency domain. Expected peak motion amplitudes in a 1-hour period are then found from the area under the response spectrum using [9, eq. 216].

11. Wave-induced motion and load results

Ship motion results from time-domain and frequency-domain calculations are shown in Figure 15 and Table 8, together with measured values. As the mooring line pre-tensions were unknown at the time of the trial, results are presented for 0.5 tonnes and 5 tonnes pre-tension in all lines, which are the likely lower and upper limits. Time-domain results are averaged over 10 runs with different input wave phasing. Standard deviation over these 10 runs was 5-15% of the mean.
Corresponding maximum predicted mooring loads are shown in Table 9. No load cells were fitted in the mooring arrangement for measuring mooring loads.

Table 9 Peak mooring line loads and fender compressions for MV Sea Diamond at 0940 on 2nd Oct 2015, according to nonlinear time-domain calculations.

<table>
<thead>
<tr>
<th>Line Type</th>
<th>TD, 0.5 tonne pre-tension</th>
<th>TD, 5 tonne pre-tension</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stern lines</td>
<td>5.2 tonnes</td>
<td>7.9 tonnes</td>
</tr>
<tr>
<td>Aft breast lines</td>
<td>12.9 tonnes</td>
<td>11.8 tonnes</td>
</tr>
<tr>
<td>Aft spring lines</td>
<td>17.4 tonnes</td>
<td>17.7 tonnes</td>
</tr>
<tr>
<td>Fwd spring lines</td>
<td>8.7 tonnes</td>
<td>10.6 tonnes</td>
</tr>
<tr>
<td>Fwd breast lines</td>
<td>12.9 tonnes</td>
<td>13.9 tonnes</td>
</tr>
<tr>
<td>Head lines</td>
<td>6.2 tonnes</td>
<td>9.5 tonnes</td>
</tr>
<tr>
<td>Fender 2 (South)</td>
<td>0.35 m</td>
<td>0.32 m</td>
</tr>
<tr>
<td>Fender 3</td>
<td>0.19 m</td>
<td>0.25 m</td>
</tr>
<tr>
<td>Fender 4 (North)</td>
<td>0.37 m</td>
<td>0.36 m</td>
</tr>
</tbody>
</table>

12. Conclusions and outlook
A coupled ship-and-harbour method has been developed for hindcasting or forecasting wave-induced motions and loads of moored ships in Geraldton Harbour. The method has been tested using measured wave spectra within the harbour, combined with GNSS measurements of a Panamax bulk carrier at Berth 5.

Future improvements to the method could include:
- Developing more accurate input far-field wave spectra, in conjunction with oceanographers
- Including harbour wall damping (if available in WAMIT in future) to calculate IRFs

Further testing of the method is desirable using load cells on shore bollards, if available in future.

13. Acknowledgements
The author would like to acknowledge:
- the pilots, harbourmaster, engineers and ship schedulers at Mid West Ports
- Karara Mining for providing support for this research
- Tremaron for supplying the harbour wave data
- Peter McComb for advice on wave spectra inside and outside Geraldton harbour
- Johan Pinkster for advice on second-order wave loads and coupled hydrodynamic coefficients
- Prof. J.N. Newman for advice and suggestions on the coupled ship-and-harbour method.

This article is dedicated to the late Martin North, Geraldton harbourmaster, and to the late Jeonghun (Scott) Ha from Curtin University, both of whom contributed to this work.

14. References