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Ship Squat in Water of Varying Depth

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SUMMARY

Two theories are described which predict ship squat in water of varying depth. Firstly, a one-dimensional theory is proposed for a ship in a narrow channel of varying depth. This is solved analytically in the case of a step depth change, and numerically for general depth profiles. Secondly, a slender-body theory is discussed for wider channels or open water. A numerical method is developed and used to find the squat of a ship passing a step depth change.

1. INTRODUCTION

Ship squat is the combined effect of midship sinkage (which is usually downward) and trim angle (which can be either by the bow or stern) caused by the hydrodynamic pressure changes beneath a moving vessel. Squat tends to decrease the underkeel clearance as the ship's speed increases (up to a certain point), and may cause a vessel to ground in shallow water, despite it having sufficient water depth at static draught.

The problem of ship squat in water of *constant* depth has been widely studied over recent decades. Most of this research has been experimental and empirical, using regression techniques to fit sinkage and trim as functions of hull and environment parameters (see e.g. [1,2,3]). Pioneering work in constant-depth theoretical squat prediction was produced by Havelock [4], for a slender ship in open water of infinite depth; also Constantine [5] for a ship in a shallow, narrow channel; Tuck [6] for a slender ship in shallow open water; and Tuck [7] for a slender ship in a shallow channel of general width. In terms of steady subcritical squat, little improvement has since been made or needed to be made on these original theories. One exception is the transcritical speed range, where nonlinear and/or dispersive terms must be added to the original equations (see e.g. [8,9]). However, these terms are relatively insignificant at the lower subcritical Froude numbers at which bulk carriers and containerships generally travel.

Interest in the problem of ship squat in *non-constant* depth has come about through the grounding of ships in shoaling water. For example, the 1992 grounding of the QE2 in Vineyard Sound, Massachusetts [10] was partly attributed to unsteady squat. In an investigation into the incident [11], researchers predicted the squat of the ship using constant-depth formulae, as there were no methods available for non-uniform depth. Then, as now, the best estimate of squat in non-uniform depth was obtained by using the average depth under the ship in the constant-depth formulae. However, it was not known whether this method would under-predict or over-predict the squat.

The extension of constant-depth theories to varying depth has to date received little attention, partly because of the difficult unsteady nature of the problem. Plotkin [12,13] solved the slender-ship problem for bottom topography that varies sinusoidally with small amplitude in the direction of travel only, by finding the resulting perturbation to the constant-depth solution. Tuck [14] discussed the extension of variable-depth theory to include depth changes transverse to the direction of travel.

Drobyshevskiy [15] put forward a slender-body method for a ship passing a step depth change, using a line of sources distributed along the step. However, the present author believes that the omission of important time-derivative terms in the quasi-steady analysis compromises those results.

Several experimental studies have been made into ship squat in varying depth, both at model scale and to a lesser extent at full scale. Model scale results were obtained by Edstrand & Norrbin [16], Ferguson et al [17], Duffield [18] and Renilson & Hatch [19]. Some of these will be discussed later with reference to the theoretical predictions. Recently, some full-scale results have been obtained using differential GPS measurements [20,21]. Although accurate, these are difficult to compare with theory due to the complex topography and non-constant ship speed.

We shall now seek to solve the problem of ship squat in non-uniform depth, for the cases of:

1. a narrow channel of arbitrary cross-sectional shape
2. open water or a channel of general width

Once the governing equations are developed, numerical algorithms for solving them will be discussed. Predictions for sinkage and trim will be given and compared qualitatively with experimental results.

2. SQUAT IN A NARROW CHANNEL

2.1 THEORY

Consider a ship travelling at constant speed U in a channel of variable cross-sectional area $S_0(x)$, where x is the earth-fixed coordinate along the axis of the channel. The waterline width of the channel is $w(x)$, and the undisturbed mean depth of the channel is $h(x) = S_0(x)/w(x)$. For this theory the depth may vary in the transverse as well as the longitudinal direction, and the ship need not be moving along the centreline of the channel, as long as it is moving in the direction of the axis of the channel.

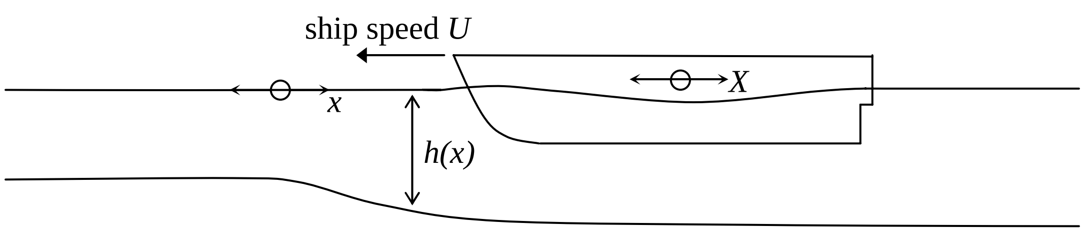

Fig. 1: Problem Formulation

In terms of the body-fixed coordinate *X*, whose origin is at midships, the ship's local beam and section area are *B(X)* and *S(X)* respectively. The earth-fixed and body-fixed coordinate systems are chosen to coincide at time *t*=0, so that they are related by $X = x + Ut$.

Linearized inviscid one-dimensional theory will be used to solve this problem. One advantage of the linearized approach is that the ship may be considered vertically fixed in its rest position for the purposes of calculating the flow field (it is inconsistent to do this if using nonlinear theory [22]).

The one-dimensional theory is valid provided that the channel is narrow and shallow compared to the ship's length, and changes in the ship and channel dimensions occur slowly in the *x*-direction [5,23]. In this case the only significant velocity component is in the *x*-direction, and this velocity is uniform across a channel cross-section. Therefore the velocity potential ϕ is a function of *x* and *t* only. The free surface height ζ is also uniform across the channel, so that the other spatial dimensions (*y* and *z*) are effectively removed from the problem.

Under these assumptions, the continuity and Bernoulli equations determine the unknowns $\zeta(x,t)$ and $\phi(x,t)$. The one-dimensional continuity equation is written in the same way as for open-channel flow [24, p.453] as

$$\frac{\partial A}{\partial t} + \frac{\partial}{\partial x}(Au) = 0 \qquad (1)$$

where *u* is the longitudinal fluid velocity ($= \phi_x$, subscripts indicating derivatives) and $A(x,t)$ is the local cross-sectional area taken up by the water. With the ship present, this is equal to the undisturbed area S_0, minus the submerged section area of the ship *S*, and adding the difference in area due to the rise or fall of the free surface. Therefore

$$A(x,t) = S_0 - S + (w - B)\zeta \qquad (2)$$

Substituting this into the continuity equation (1) gives the unsteady continuity equation for flow past a ship in a channel. Note that since $S = S(X)$ and $X = x + Ut$, the *x* and *t* derivatives become

$$\frac{\partial S}{\partial x} = S'(X)$$

$$\frac{\partial S}{\partial t} = U S'(X)$$

where the primes denote differentiation with respect to the body-fixed coordinate *X*. *B*(*X*) is differentiated in the same manner.

For ship and channel configurations in which S / S_0 and B / w are small quantities, *u*, ϕ_t and ζ are all small quantities, so that we may neglect terms of second order in any of these quantities. In this case the linearized version of equations (1) and (2) becomes

$$-U S'(X) + w\zeta_t + (uS_0)_x = 0 \qquad (3)$$

The other equation to be used is the Bernoulli equation applied on the free surface. Written in earth-fixed coordinates, this becomes

$$g\zeta + \tfrac{1}{2}u^2 + \phi_t = 0$$

which linearizes to

$$g\zeta + \phi_t = 0$$

This, combined with the continuity equation (3) to eliminate ζ, results in the equation for linearized unsteady one-dimensional flow around a ship in a non-uniform channel:

$$\frac{\phi_{tt}}{g} - \frac{1}{w}(S_0\phi_x)_x = -\frac{US'(X)}{w} \quad (4)$$

In the case where the channel is of constant width *w* but variable depth *h*(*x*), this simplifies to

$$\frac{\phi_{tt}}{g} - (h\phi_x)_x = -\frac{US'(X)}{w} \quad (5)$$

This result was given in [25], with the velocity potential in that article scaled with respect to *U*. It is a one-dimensional wave equation with a forcing term. Therefore in an unsteady situation, such as when a ship is passing depth changes, one-dimensional travelling waves may be radiated away from the ship. When a ship is in constant depth, the flow is steady, with no radiated waves.

The boundary conditions for equation (5) are that there should be no disturbance far from the ship, i.e. that ϕ_x and ϕ_t should both vanish as $x \to \pm\infty$.

Once equation (4) or (5) is solved for the velocity field, the pressure on the ship's hull can be found using Bernoulli's equation. In terms of the hydrodynamic pressure *p*, which is the excess above hydrostatic pressure, Bernoulli's equation

$$p + \tfrac{1}{2}\rho\phi_x^2 + \rho\phi_t = 0$$

applies throughout the fluid. Therefore the hydrodynamic pressure is given to first order as

$$p = -\rho\phi_t$$

in the earth-fixed coordinate system. This pressure is integrated over the ship's hull at each instant in time to obtain the time-varying heave force and pitching moment on the ship. Hydrostatic balancing then allows the time-varying sinkage and trim to be determined in the same way as for steady flow [6].

It is appreciated that the time-dependent force and moment will tend to produce dynamical oscillations in the heave and trim as well as overall changes. However, the natural period of both heave and pitch is generally significantly less than the time taken for the ship to experience depth changes. For example, a 300-metre ship will generally have natural heave and pitch frequencies of around 8-10 seconds [26]. Even for the drastic case of this ship passing a step depth change at 20 knots, the time taken for this to occur (30 seconds) is significantly larger than the natural heave and pitch periods. Therefore we may expect that dynamic effects will be small, and that hydrostatic balancing should closely estimate the mean sinkage and trim. We also expect that the amplitude of any superimposed dynamical oscillations will be small.

2.2 ANALYTIC SOLUTION FOR A STEP DEPTH CHANGE

Equation (5) has an exact analytic solution in the case of a ship passing a step depth change in a channel of constant width. This is given in [25] with more detail shown in [23]. This shall be used as a test of the general numerical method.

Note that one of the assumptions of one-dimensional theory, that of slowly-varying channel dimensions, is actually violated at the step. However, Lamb [27, p. 262] showed that, provided the flux $wh\phi_x$ and free surface height ζ are continuous at the step, open channel flow will still be accurate outside the immediate vicinity of the step. The same analysis shows that one-dimensional flow will also still be applicable for flow past a ship. Therefore the error, due to violation of the slowly-varying assumption, only occurs in a small region close to the step, and the theory may justifiably be used for overall quantities such as sinkage and trim.

2.3 NUMERICAL METHOD

A finite-difference method will be outlined here for solving the governing equation (5) for general depth profiles, in the case of a constant-width channel.

Because of the first derivative term $h_x\phi_x$, centred-difference schemes to solve for ϕ are unstable according to the Neumann criterion [28, p.155]. To overcome this difficulty, we differentiate equation (5) with respect to *x* and write it in the form

$$(h\phi_x)_{tt} - gh(h\phi_x)_{xx} = -\frac{gUh}{w} S''(X) \qquad (6)$$

In this way we can use centred differencing to solve instead for $h\phi_x$, which gives the time-stepping routine

$$f_j^{n+1} = (2-2s_j) f_j^n - f_j^{n-1} + s_j (f_{j+1}^n + f_{j-1}^n) - \frac{gUh_j(\delta t)^2}{w} S''(x_j + Ut_n)$$

where $f = h\phi_x$ and $f_j^n = f(x_j, t_n)$. δt is the time step, s_j is the parameter $gh_j(\delta t/\delta x)^2$ and $h_j = h(x_j)$. This procedure is numerically stable, according to the Neumann criterion, provided that $s_j \leq 1$ throughout the computational domain.

In order to avoid numerical reflection from the boundaries, the computational domain is chosen to be large enough that waves cannot propagate to the boundaries by the time the final time step is reached. Therefore our boundary conditions are that $f = 0$ at the upstream and downstream boundaries.

In the case where the ship starts in water of constant depth, the steady-state solution [7]

$$f(x,t) = \frac{US(x+Ut)}{w(1-F_h^2)} \qquad (7)$$

can be used as an initial condition at the first two time levels. Here F_h is the depth-based Froude number $F_h = U/\sqrt{gh}$. In the case where the ship starts in non-uniform depth, the above expression (with $h = h(x)$ in the definition of F_h) provides a reasonable quasi-steady estimate of the initial flow.

Because the governing equation (6) is a wave equation, the finite-difference method is also prone to producing spurious travelling waves radiating away from the ship's bow and stern. In order to minimize these, a fore-aft-symmetric "cusped" hull was considered, which has a very fine bow and stern at which $S'' = 0$. In terms of the shiplength L and maximum beam and section area $B_{\max}$ and $S_{\max}$ respectively, the hull's local beam and section area are defined by

$$B(X) = B_{\max}\left[1-\left(\frac{2X}{L}\right)^2\right]$$

$$S(X) = S_{\max}\left[1-\left(\frac{2X}{L}\right)^2\right]^3$$

The radiating waves were also minimized by choosing a small grid interval δx. However, this increased the problem of numerical drift [23], so a compromise had to be reached in order to obtain accurate results.

2.4 RESULTS FOR A STEP DEPTH CHANGE

The numerical results agreed with the analytic solution previously discussed. Further results will be presented here, for the case of a ship passing a step depth change into shallower water. We shall only be considering the scenario in which the flow is *subcritical* on both sides of the step (i.e. $F_h < 1$ in both regions). Although the analytic and numerical methods are both still valid for *supercritical* flow, the validity of one-dimensional theory itself is questionable in this case [23, p.13], besides which the fully-subcritical case is by far the most common in practice.

As discussed in the preliminary results of Gourlay & Tuck [25], the passage of the ship past the step results in two travelling waves, each radiating out from the step in opposite directions.

These waves start being formed when the ship's bow passes the step and are completed when the ship's stern passes the step. They then radiate out to infinity (in the absence of dissipation). While the ship is passing over the step, the waves act to *decrease* the flow speed past the ship. Even once the ship's stern has passed the step, the flow speed past the hull is still affected until the wave travelling in $x < 0$ completely overtakes the ship.

In the case that the ship is passing a step depth change into deeper water the opposite occurs – the velocity waves act to *increase* the flow speed past the ship.

The effect of these waves on the midship sinkage is shown in Fig. 2. Because the ship is fore-aft symmetric, the vertical force on the ship in its rest position (Z) translates directly into midship sinkage (s) through the relation

$$Z = -\rho g A_W s$$

where A_W is the waterplane area. Therefore Fig. 2 represents both the scaled vertical force and scaled midship sinkage. For these results the ratio between the two depths is 0.75, the channel's width is the same as the initial depth, and the depth-based Froude number is equal to 0.143 in the deeper water. The vertical axis is scaled such that the results are valid for any values of the parameters having the ratios described above.

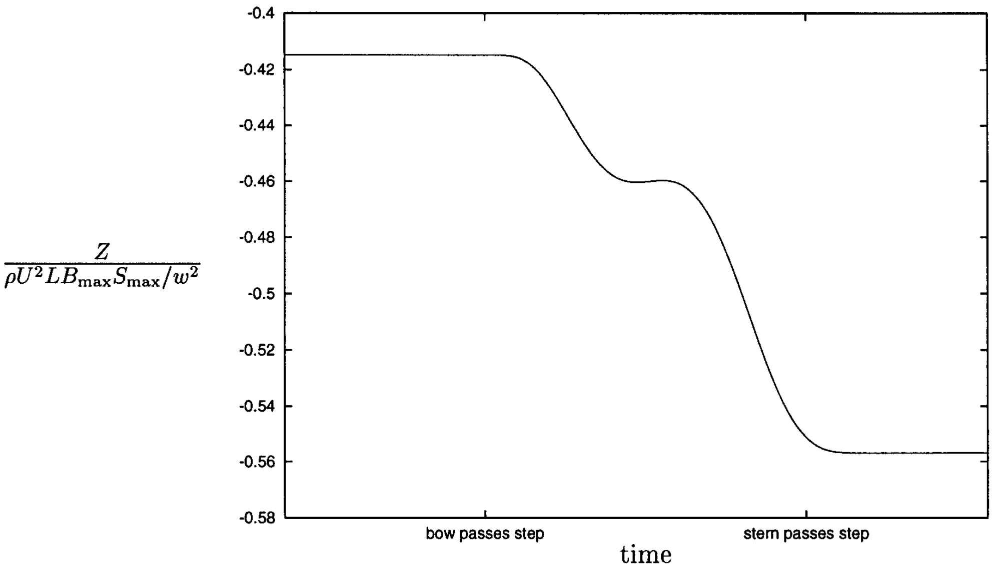

Fig. 2: Scaled vertical force or midship sinkage for a ship passing a step depth change to shallower water in a narrow channel

It is seen that, rather than moving smoothly from the deep-water to the shallow-water steady state value, there is a lag which results from the flow retardation due to the two waves. This means that the vertical force does not reach the steady state value until after the ship's stern passes the step; also, it at no time "overshoots" the shallow-water value.

The transient trim moment is shown in Fig. 3. The vertical axis is again scaled such that the results for this hull shape are valid for any values of the parameters for which the ratio between the two depths is 0.75, the channel's width is the same as the initial depth, and the depth-based Froude number is equal to 0.143 in the deeper water.

For a fore-aft-symmetric ship, the trim moment M is related hydrostatically to the trim angle through the relation

$$M = \rho g I_W \theta$$

where θ is the bow-up trim angle and

$$I_W = \int_{-\frac{L}{2}}^{\frac{L}{2}} X^2 B(X)\, dX$$

Therefore Fig. 3 also shows the bow-up trim angle, scaled appropriately. We see that, apart from a small bow-down trim (i.e. trim by the bow) as the bow passes the step, the trim is strongly bow-up (i.e. trim by the stern) while the ship is passing the step. This bow-up trim continues after the stern passes the step.

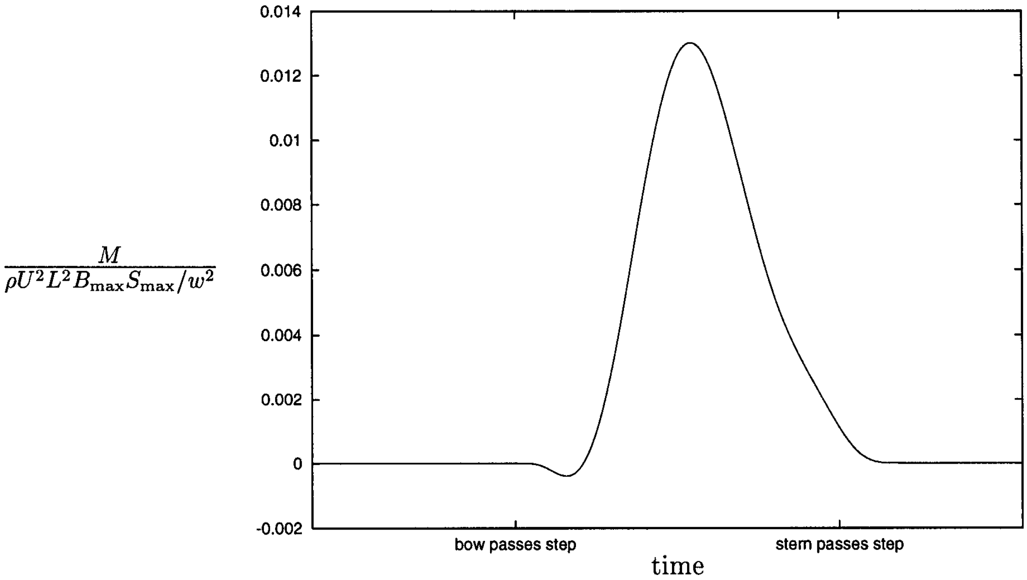

Fig. 3: Scaled bow-up trim moment or trim angle for a ship passing a step depth change to shallower water in a narrow channel

The results for midship sinkage and trim can be easily combined to give the transient bow and stern sinkage, which are shown in Fig. 4.

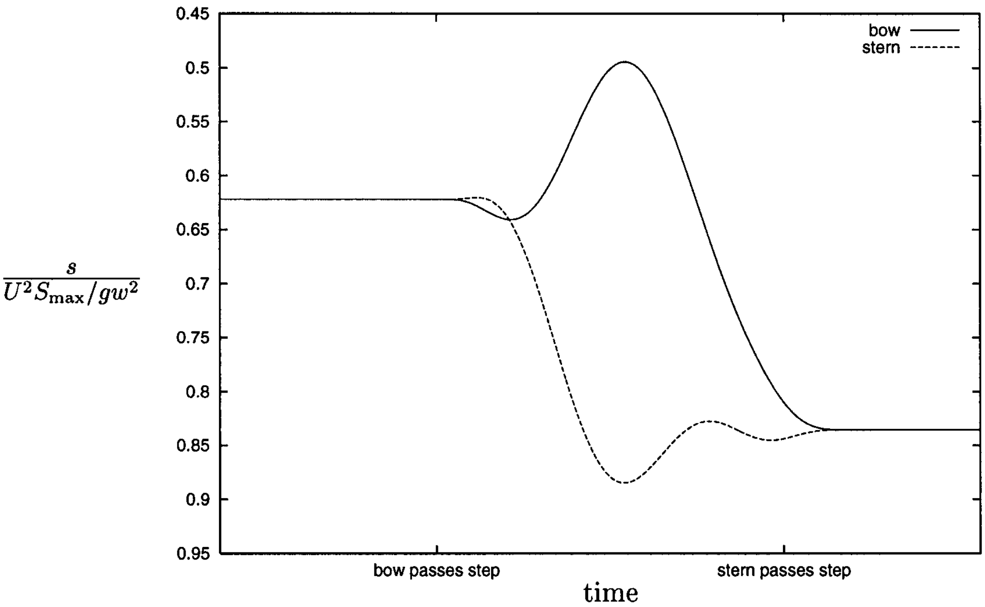

Fig. 4: Bow and stern sinkage for a ship passing a step depth change to shallower water in a narrow channel

For the case of a ship passing a step depth change to *deeper* water, the calculated vertical force and trim moment are almost the mirror image of the deep-to-shallow case. That is, the ship experiences a bow-down trim while passing into the deeper water, and the midship sinkage moves slowly to the steady-state value.

2.5 RESULTS FOR A SMOOTHLY SHELVING SEA FLOOR

Numerical results were obtained for the case of a channel whose floor slopes at a constant angle along the axis of the channel. In this case it was found that the flow field was indistinguishable from the "quasi-steady" approximation given by equation (7), with $h = h(x)$ in $F_h = U/\sqrt{gh}$.

2.6 RESULTS FOR A RAMP BETWEEN TWO CONSTANT DEPTHS

The transient midship sinkage and trim for a ship passing a ramp between two constant depths are shown in Figs. 5 and 6. The quasi-steady approximations obtained using equation (7) are also shown for comparison. It is seen that the midship sinkage changes almost linearly between the two steady-state values, while the trim remains bow-up over the entire length of the ramp.

The quasi-steady approximation provides a good estimate to the full unsteady solution, giving very similar sinkage and slightly under-predicting the trim.

The ramp for which results are shown is three times the length of the ship; shorter ramps produced larger bow-up trim angles, and longer ramps smaller bow-up trim angles.

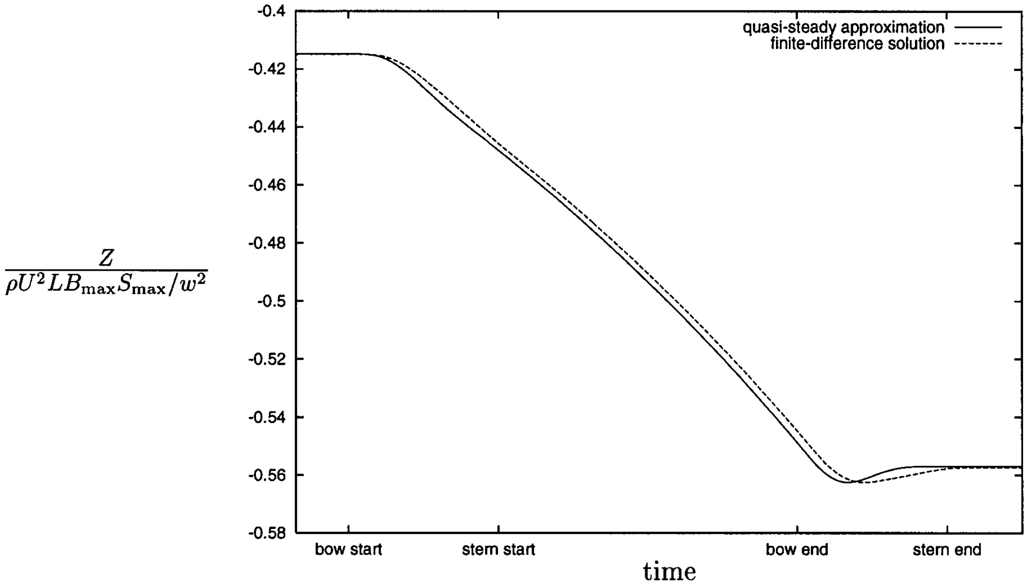

Fig. 5: Scaled vertical force or midship sinkage for a ship passing a ramp between two constant depths in a narrow channel. Plotted as a function of time, with the points at which the bow or stern passes the start or end of the ramp indicated.

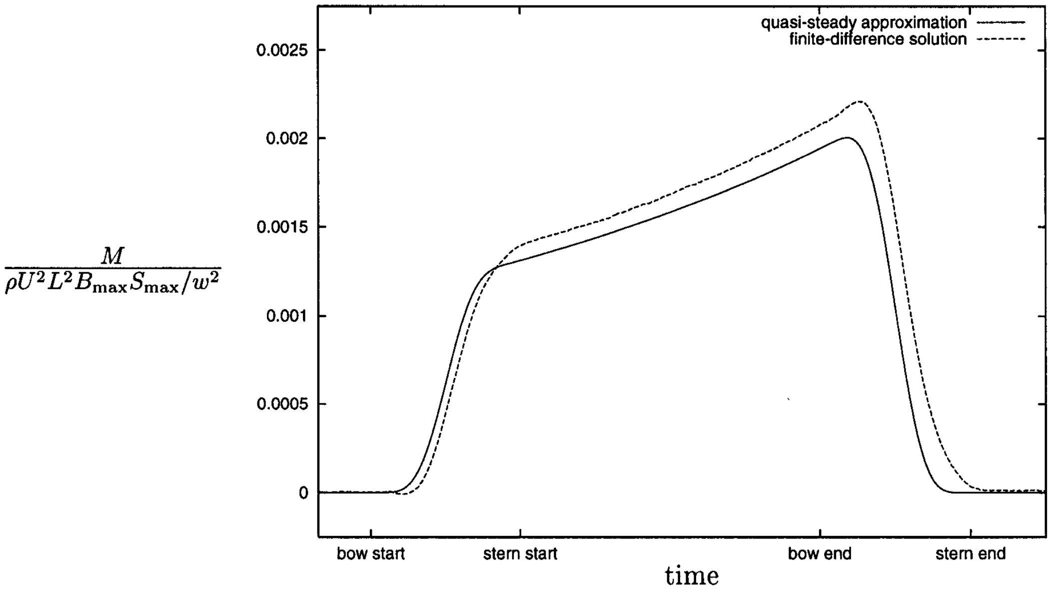

Fig. 6: Scaled bow-up trim moment or trim angle for a ship passing a ramp between two constant depths in a narrow channel. Plotted as a function of time, with the points at which the bow or stern passes the start or end of the ramp indicated.

3. SQUAT IN OPEN WATER OR A CHANNEL OF GENERAL WIDTH

3.1 THEORY

For this problem, slender-body inviscid theory will be used. The flow is divided into two regions: an "outer" region far from the ship, and an "inner" region close to the ship. Governing equations are found for the flow in each of these regions, starting with Laplace's equation and the kinematic and Bernoulli boundary conditions. The two flows are then matched across the interface. The method is a simple extension of the constant-depth approach developed by Tuck [6].

The coordinate system used is the same as for the narrow-channel case, except that we also define the vertical coordinate (z, centred at the still water line) and transverse coordinate (y, centred at the ship's centreline). The depth is allowed to vary in both horizontal directions, so that $h = h(x, y)$.

In the *outer* region, the shallow-water assumption states that the vertical length scale is small compared to the horizontal length scales (which may be the ship length or wavelength). The resulting asymptotic analysis shows that, to leading order, the velocity potential is independent of z and satisfies the equation

$$\frac{\phi_{tt}}{g} = (h\phi_x)_x + (h\phi_y)_y \qquad (8)$$

in the earth-fixed coordinate system. The equivalent version of this equation was obtained by Plotkin [12] in body-fixed coordinates. The earth-fixed equation above is oblivious to the presence of the ship, and is the same as that given by Stoker [24] to describe long waves propagating in shallow water.

In the *inner* region, the length scales in both the *y* and *z* directions are small compared to that in the *x* direction. As such, Laplace's equation and the boundary conditions effectively revert to a two-dimensional problem in the *y*-*z* plane. The inner flow is driven by the changing cross-sectional area *S* of the ship's hull, which causes water to be pushed aside when *S* is increasing with *X*, and pulled in when *S* is decreasing with *X*. The behaviour of this inner flow further from the hull provides the boundary condition which drives the outer flow.

Reanalysis of the inner flow for water of non-uniform depth shows that the method is still valid when depth changes occur in the *x*-direction.

If the bottom slope is not large compared to *h/L*, then the depth is effectively constant over the whole of the inner region. Therefore if only *symmetric* depth changes occur in the *y*-direction, i.e. $h(x,y) = h(x,-y)$, and the channel walls are symmetric about the ship's centreline, then the matching process goes over unchanged from the constant-depth case.

That is, the resulting boundary condition for the outer equation (8) becomes

$$\phi_y(x,0_\pm,t) = \pm\frac{U\,S'(x+Ut)}{2\,h(x,0)} \qquad (9)$$

This is an extension of the result of Tuck [6] in which the constant depth *h* is replaced by $h(x,0)$.

If *non-symmetric* depth changes occur in the *y*-direction, the inner flow may also include a cross-flow, and the matching process becomes more complicated. Further research is required in this area.

In the derivation of this theory, no assumptions have been made about the channel width. In fact, the theory is equally valid for open water or for very narrow channels; in the latter case it becomes equivalent [23] to the one-dimensional theory previously described.

3.2 GENERAL NUMERICAL METHOD

The partial differential equation (8), along with the boundary condition (9), can be solved using finite-difference methods. As in the one-dimensional case, the equation must be solved in a form that will be numerically stable. This can be achieved by writing the equation in the form

$$\frac{(h\phi)_{tt}}{g} = h\nabla^2(h\phi) - (h\phi)\nabla^2 h$$

and solving for the quantity $(h\phi)$. Clearly this method is more suited to smooth depth changes because of the $\nabla^2 h$ term.

The boundary conditions on this equation in *open water* are that the derivatives of ϕ either vanish or behave like an outgoing wave far from the ship. If the finite-difference equation is being solved in open water, artificial boundaries must be used at a sufficient distance from the ship. These boundaries may only be within the domain of influence of the ship if non-reflecting boundary conditions are successfully applied. Otherwise, all ϕ-derivatives can be set to zero if the boundaries lie outside the domain of influence of the ship.

For the case of a ship in a channel, the size of the computational domain in the *y*-direction is limited. In this case we specify the boundary condition of zero fluid velocity normal to the channel walls.

In a similar manner to the one-dimensional theory previously described, the constant depth solution can be used as an initial condition. This is given in [6] as

$$\phi(x,y,t) = \frac{U}{2\pi h\sqrt{1-F_h^2}} \int_{-\frac{L}{2}}^{\frac{L}{2}} S'(\xi) \log_e \sqrt{(x+Ut-\xi)^2 + (1-F_h^2)y^2}\, d\xi$$

If the ship is in water of non-uniform depth at time *t*=0, a quasi-steady estimate can be obtained by replacing *h* by the local value $h(x,0)$ in this expression. Note that F_h is also non-constant in this approximation.

Once the velocity potential is solved for at each time step, matching of the inner and outer flows shows that the hull pressure is given in earth-fixed coordinates by

$$p = -\rho\phi_t(x,0,t)$$

to leading order. The instantaneous vertical force, moment, sinkage and trim can then be found at each time step in the same way as for the one-dimensional theory.

3.3 NUMERICAL METHOD FOR A STEP DEPTH CHANGE IN A CHANNEL

Let us consider the problem of a ship moving along the centreline of a channel and passing a step depth change in the *x*-direction. In this case the flow is symmetric about *y*=0, so we only need consider the region $y > 0$. In each region of constant depth, the partial differential equation (8) becomes

$$\phi_{tt} = gh(\phi_{xx} + \phi_{yy})$$

which is an ordinary wave equation in two dimensions.

This equation can be centre-differenced to second order, to obtain a time-stepping routine for ϕ in each region. As for the one-dimensional theory, the quantities $h\phi_x$ and ϕ_t are then required to be continuous across the step. By positioning gridpoints along the step, i.e. common to both regions, the continuity of ϕ and therefore of ϕ_t are assured. The continuity of $h\phi_x$ is imposed by equating its finite-difference forms in each region.

The wall and centreline boundary conditions are enforced by positioning a line of gridpoints along each of the boundaries and an extra row of fictitious gridpoints outside. In this way the finite-difference method can be applied on the boundaries, and the boundary conditions used to eliminate the fictitious gridpoints. This gives a consistent second-order solution throughout the domain.

Since this slender-body theory agrees with the one-dimensional theory in the narrow-channel limit, one-dimensional waves are produced when the ship passes the step in a narrow channel. In fact, numerical simulations show that this is still true for quite wide channels. Therefore, instead of using a very large computational domain in the x-direction and setting all ϕ-derivatives to zero on the upstream and downstream boundaries, we instead use a moderate computational domain and the non-reflecting boundary conditions

$$\phi_t + \sqrt{gh}\,\phi_x = 0 \quad \text{in } x > 0$$

$$\phi_t - \sqrt{gh}\,\phi_x = 0 \quad \text{in } x < 0$$

This allows any one-dimensional waves produced to pass through the edges of the computational domain without reflection.

With this method, the same problems of numerical wave radiation, numerical oscillation and drift were encountered as in the one-dimensional case. However, drift proved harder to eliminate in two dimensions.

3.4 RESULTS

Simulations were performed for various channel width to shiplength ratios. It was seen that for narrow channels the flow field was almost identical to the one-dimensional theory, with one-dimensional waves being produced as the ship passed the step. For wider channels, the flow velocities were significantly larger close to the ship, as the steady theory predicts. However, the radiated waves, though very small in amplitude, were still completely one-dimensional.

In Fig. 7 we see the form of the transient vertical force on a ship passing a step depth change to shallower water in a wide channel. Again, this is also proportional to the midship sinkage. In this case the channel width is twice the shiplength, the ratio between the two depths is 0.75, and the depth-based Froude number is equal to 0.143 in the deeper water. We have used arbitrary values of the other ship parameters, as the results cannot be scaled as simply as for the one-dimensional theory.

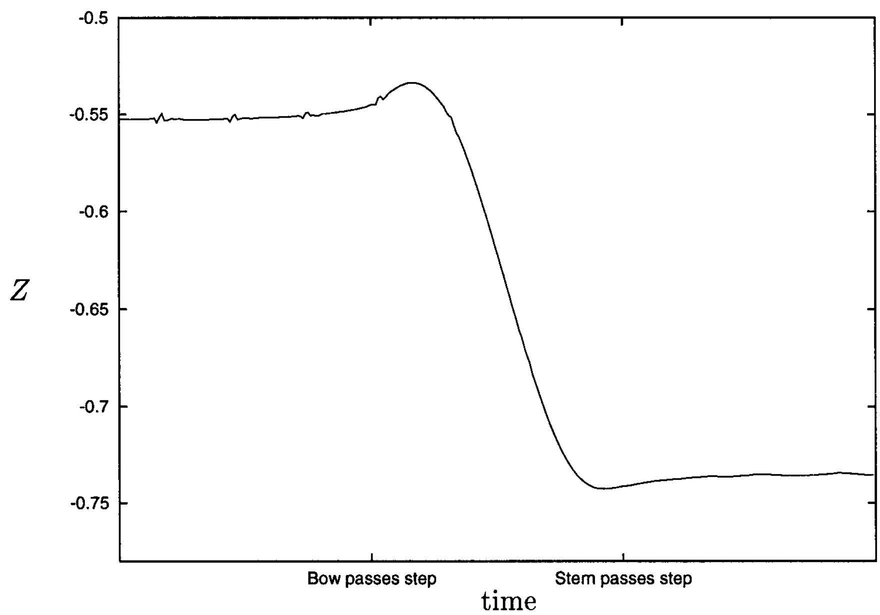

Fig. 7: Vertical force or scaled midship sinkage for a ship passing a step depth change to shallower water in a wide channel.

It is seen that the results are quite different than for a narrow channel in the transition time between the two steady state values. In this case the ship *does* sense the presence of the step before the bow reaches it, with a smaller midship sinkage initially. The effect of the radiated waves is not apparent in this case, since these are negligible at this large channel width. Instead the sinkage moves smoothly down and actually overshoots the steady shallow-water value.

The small recurring spikes are due to irregularities in the initial conditions. Without any damping mechanism, these radiate out to the channel walls and reflect back periodically to affect the flow on $y = 0$.

Fig. 8 shows the transient bow-up trim moment for the same case. Again, this is also proportional to the transient trim angle. Note that the hull is fore-aft symmetric, so that the trim should be zero according to the theory when the ship is not near the step. The small residual trim is due to numerical drift. Nevertheless, there is a definite bow-up trim while the ship is passing the step.

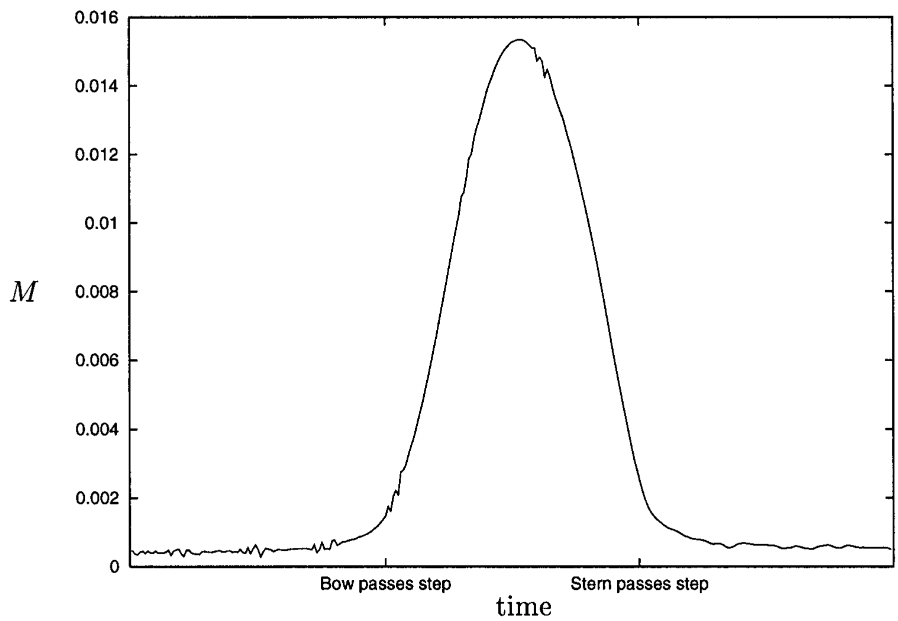

Fig. 8: Bow-up trim moment or scaled trim angle for a ship passing a step depth change to shallower water in a wide channel.

It can be seen that the ship's trim also changes before the bow reaches the step. For the symmetric ship, this results in a small bow-up trim both before and after the ship passes the step. It is unknown at this stage how the trim will vary for a more realistic non-fore-aft-symmetric ship.

The combined effect of the midship sinkage and trim angle gives the bow and stern sinkage, or squat. Fig. 9 shows qualitatively how these change between the two steady-state values for our cusped hull.

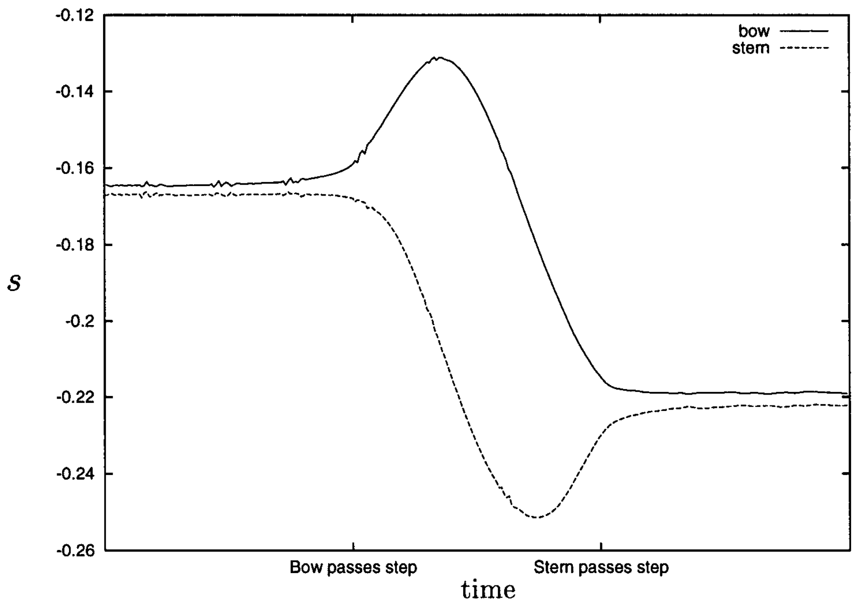

Fig. 9: Bow and stern sinkage for a ship passing a step depth change to shallower water in a wide channel.

The small residual difference between the bow and stern sinkages is due to the residual trim angle and can be ignored. The results have a similar form to the narrow channel case (Fig. 4), except that a more dangerous stern sinkage occurs while the ship is passing the step.

4. QUALITATIVE COMPARISON WITH EXPERIMENT

An experimental investigation into squat in non-constant depth (in which the author participated) was undertaken by Duffield [18]. Experiments were performed with a bulk carrier and a containership, for a step depth change, ramp depth change, or short shallow bank. The experiments were performed in a channel whose width was twice the ship length, such that slender-body theory would be expected to be more accurate than one-dimensional theory in this case.

Some of the experimental results are described qualitatively below, with reference to the results of the theoretical treatment described here. Note that at this stage the theoretical results are only for a simple fore-aft symmetric ship, rather than the exact ship shapes.

4.1 STEP DEPTH CHANGE TO SHALLOWER WATER

For the bulk carrier, the model increased its bow-down trim while approaching the step, before rapidly changing to a bow-up trim angle while passing over the step. The bow-up trim is in qualitative agreement with the theories for a fore-aft symmetric ship. The bow-up trim is clearly a more "forgiving" situation for a ship passing over a step depth change to shallower water, as it will decrease the likelihood of grounding.

The experimental sinkage was unexpected: the midship sinkage increased slightly before the ship reached the step, whereas slender-body theory predicts it to decrease slightly. After the ship passed the step, the sinkage did not immediately reach the new steady state, as the slender-body theory predicts. The experimental lag in reaching the new steady state was witnessed in both the sinkage (for free-to-squat experiments) *and* the vertical force (for vertically fixed experiments). It was seen consistently in all experiments, both for the step and the ramp, and was perhaps the most interesting and surprising aspect of the testing. Clearly, a lag in reaching the new steady state sinkage (which is greater in shallow water) is a more desirable outcome, as it will decrease the likelihood of grounding.

The author presently believes that the lag may be due to viscous effects in the thin layer of water between the hull and the sea floor, which may retard the time-evolution of pressure changes beneath the hull. Further research is required in this area.

4.2 RAMP DEPTH CHANGE TO SHALLOWER WATER

The length of the ramp was roughly 40% longer than the length of the ship models.

For the bulk carrier, the trim was slightly bow-down while the model was passing the ramp, before rapidly changing to bow-up as the bow passed the end of the ramp. These effects were also witnessed consistently by Ferguson et al [17] during their experiments with a bulk carrier passing a ramp.

By comparison, the one-dimensional theory for a fore-aft symmetric hull predicts a bow-up trim over the entire ramp. It is not known at this stage how this result will be modified for a non-fore-aft-symmetric hull. The observed bow-down trim is a dangerous phenomenon, which Ferguson et al [17] concluded may have contributed to the grounding of the MV Wellpark in La Plata Estuary in 1977. The present author believes that this effect may be due to large local fluid velocities beneath the forward section of the bulk carrier, due to its fullness, causing low pressures and a corresponding bow-down trim.

For the containership, no such bow-down trim was witnessed. The trim was strongly and consistently bow-up over the entire length of the ramp, as predicted by the theory for a fore-aft symmetric hull.

5. CONCLUSIONS

A one-dimensional theory has been proposed for studying the squat of a ship in a narrow channel of varying depth. This has an analytic solution for a step depth change, and can be solved numerically for general depth profiles using a finite-difference scheme.

A slender-body theory has been discussed which covers two-dimensional depth changes, and a numerical scheme introduced for solving the flow field in the general case. This slender-body theory reduces to the one-dimensional theory in the narrow-channel limit.

Results were computed for the case of a cusped fore-aft symmetric ship passing a step depth change, according to both theories. The one-dimensional theory was also used for a ship in uniformly-shoaling water, or passing a ramp between two constant depths.

There were several numerical issues which made solution of both theories difficult. Artificial numerical wave production was the major problem, which meant that at this stage accurate results could only be given for sharp-ended vessels.

Nevertheless, some important unsteady effects were witnessed in the results, such as the strong bow-up trim experienced by a fore-aft symmetric ship passing a step or ramp depth change to shallower water. These agreed qualitatively with experimental research.

Further research is now required to extend the numerical method to handle more realistic hull shapes without numerical wave production, so as to study dangerous situations such as full-form vessels passing a ramp depth change to shallower water.

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