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The Effect of Squat on Steady Nonlinear Hydraulic Flow Past a Ship in a Channel

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Abstract

We present a method for computing steady nonlinear hydraulic flow past a ship in a channel. This method recognizes the fact that allowing the ship to sink and trim will give a different flow from that obtained by fixing the ship vertically in its rest position. Actual sinkage and trim, as well as limiting Froude numbers for steady flow, are computed.

1 Introduction

For a slender ship whose beam and draught are small compared to the width and depth respectively of the channel in which it is travelling, the flow velocity perturbations due to the presence of the ship are small, and the flow may be linearized. Sinkage and trim can then be calculated by first solving the flow field with the ship assumed unable to move vertically.

The ship sinks and trims by a small amount such that the hydrostatic nett vertical force and moment on the ship vanish [7]. This assumes that the flow field will change negligibly when the ship is allowed to squat.

In linearized flow this is an acceptable approximation, because the difference in flow velocities betweeen a ship fixed in its equilibrium position and the same ship allowed to squat is small, of the same order as other terms already neglected.

However, if the ship's beam and draught are comparable with the width and depth respectively of the channel, flow velocities are no longer small and we cannot make this approximation. The flow field must be solved as a function of the ship's sinkage and trim. As these quantities are unknown in advance, a set of nonlinear simultaneous equations will result.

Although the problem is solved here using one-dimensional hydraulic theory, the method for calculating sinkage and trim is applicable to any nonlinear theory for ship dynamics.

2 Problem Formulation

For simplicity we take the ship to be moving at constant speed U, down the middle of a rectangular channel of constant width w and constant undisturbed depth h. In fact, the theory may be applied to channels of arbitrary cross-section, provided that they are vertical-sided near the waterline. In that case h is the average depth across the channel, and w is the waterline channel width.

We consider a frame of reference that is fixed relative to the ship, such that the coordinate X is in the direction of the free stream. The ship's bow is at $X = -\ell$ and stern at $X = \ell$, where 2ℓ is the shiplength.

The ship's local waterline beam and local cross-sectional area at position X are $B(X)$ and $S(X)$ respectively, both of which are defined on $[-\ell, \ell]$.

3 The Hydraulic Equations

In hydraulic theory [2], it is assumed that changes to the free-stream velocity are greatest in the direction of the free stream, so that other velocity components may be neglected. This is justified if S and B are slowly-varying functions of X, and the channel is narrow and shallow compared to the ship's length. It also assumes that the streamwise velocity is uniform across the channel, which follows from the first assumptions by irrotationality.

We will denote the streamwise flow speed as γU, where $\gamma = \gamma(X)$ is constant across any particular cross-section $X =$constant. The height of the free surface above the undisturbed level is defined as βh. Similarly, from the previous assumptions, $\beta = \beta(X)$ is constant across the whole width of the channel. Neither $\gamma - 1$ nor β are small in general.

The squat of the ship is measured by its midships sinkage sh and its bow-up angle of trim θ. Therefore the downward displacement of any station X on the ship is $sh + X \tan\theta$, which we will define as σh.

The equation of continuity for steady one-dimensional hydraulic flow is obtained by matching the flux across any cross-section $X =$ constant to the flux far upstream. This gives

$$\gamma U A(X) = U S_0$$

where $S_0 = wh$ is the cross-sectional area taken up by the undisturbed water upstream and $A(X)$ is the local cross-sectional area taken up by the water as it passes the ship. This area has the form

$$A(X) = S_0 - S + (w - B)\beta h - B\sigma h$$

where the $B\sigma h$ term gives the decrease in area available due to the local sinkage σh. We have assumed here that the ship and channel walls are vertical at the waterline. The continuity equation then simplifies to

$$\gamma \left[1 - \frac{S}{S_0} - \sigma\frac{B}{w} + \left(1 - \frac{B}{w}\right)\beta\right] = 1 \qquad (1)$$

The Bernoulli equation applied on the free surface gives

$$\frac{1}{2}(\gamma U)^2 + g(\beta h) = \frac{1}{2}U^2$$

where g is the acceleration due to gravity. This can be simplified to

$$\gamma^2 + \frac{2\beta}{F_h^2} = 1 \tag{2}$$

where F_h is the depth-based Froude number $F_h = U/\sqrt{gh}$.

If $\sigma(X)$ is known, the continuity equation (1) and Bernoulli equation (2) constitute two simultaneous equations which can be solved for the unknowns β and γ at each value of X. Specifically, eliminating γ gives the following cubic equation for the scaled free surface height β:

$$\begin{aligned} &\frac{2}{F_h^2}\left(1-\frac{B}{w}\right)^2\beta^3 + \left[\frac{4}{F_h^2}\left(1-\frac{S}{S_0}-\sigma\frac{B}{w}\right)\left(1-\frac{B}{w}\right)-\left(1-\frac{B}{w}\right)^2\right]\beta^2 \\ +\ &\left[\frac{2}{F_h^2}\left(1-\frac{S}{S_0}-\sigma\frac{B}{w}\right)^2 - 2\left(1-\frac{S}{S_0}-\sigma\frac{B}{w}\right)\left(1-\frac{B}{w}\right)\right]\beta - \left(1-\frac{S}{S_0}-\sigma\frac{B}{w}\right)^2 + 1 = 0 \end{aligned} \tag{3}$$

The cubic equation (3) can have between one and three real roots, so we must be careful to choose the correct root.

The real root which is always present represents a backflow ($\gamma < 0$) of negative cross-sectional area. This is unphysical and inadmissible. In order to find the correct solution, we can plot the continuity relation (1) and Bernoulli equation (2) for various values of F_h, and look at the intersection points. This is depicted in Fig. 1, with sample values for B/w, S/S_0 and σ having been chosen. Only the physically admissible branch $\gamma > 0$ is shown.

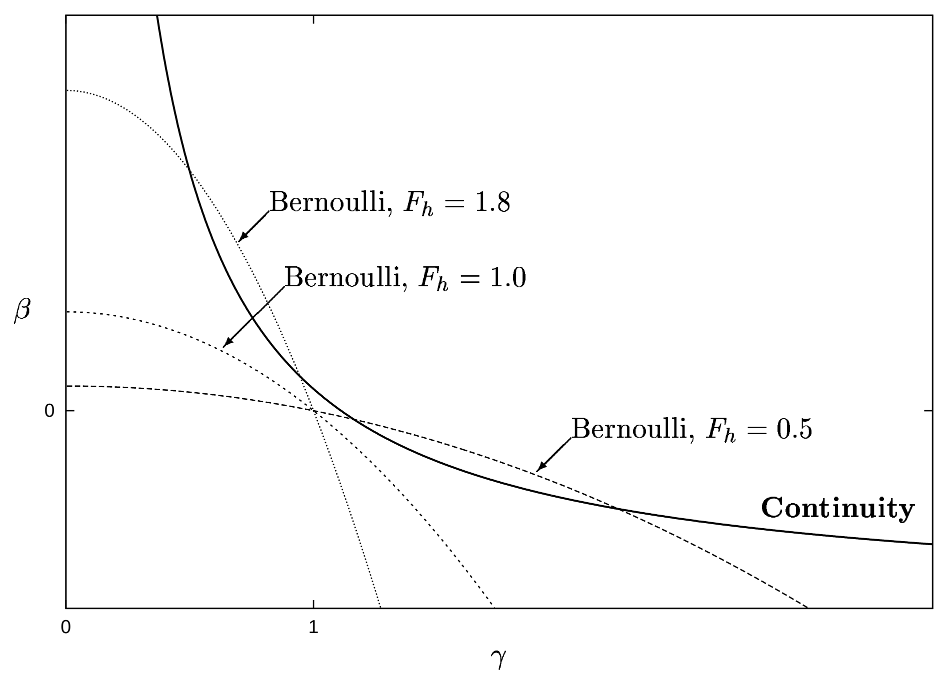

Figure 1: Typical continuity and Bernoulli relations between scaled free surface height β and scaled velocity γ

We see that at low F_h (e.g. F_h=0.5 in Fig. 1) there are two intersection points in the region $\gamma > 0$, both of which have $\gamma > 1$. One of these represents a greatly accelerated flow past the ship, with a greatly depressed free surface. Considering small changes in the free surface height, with continuity still satisfied, we can show by surface pressure arguments that this flow is statically unstable. The other intersection point, which represents a slightly accelerated flow with a slightly depressed free surface, is statically stable and the root we require. This is termed "subcritical" flow.

The two intersection points correspond to the "hydraulic jump" observed in channels [6]. This is a sudden transition from a high speed, low free surface height flow to a low speed, large free surface height flow. Although both of these flow regimes satisfy the Bernoulli and continuity conditions, the latter only occurs downstream, with energy being lost across the transition. This is a special case of the problem solved here, in which no ship is present so that B/w and S/S_0 are zero.

For intermediate values of F_h (e.g. F_h=1.0 in Fig. 1) there are no intersection points with $\gamma > 0$. This is the "critical" region in which no steady hydraulic flow exists, a situation which will be discussed in Section 6.

When F_h is large (e.g. F_h=1.8 in Fig. 1) there are again two intersection points in the region $\gamma > 0$. In this case, however, both roots have $\gamma < 1$. As for subcritical flow, we can show using surface pressure arguments that only the root with the higher free surface and lower speed is stable in a static sense. This 'supercritical" flow which we shall consider is therefore significantly decelerated, with a large free surface height. It is of less practical interest than subcritical flow for ships in narrow channels, as a very high speed is normally required for steady supercritical flow to exist.

So if $\sigma(X)$ is known, we can determine the correct profile of $\beta(X)$ using (3). However, $\sigma(X)$ is not known in advance; the two equations which determine its two parameters s, θ are the vertical force and trim moment equilibrium equations. Since for hydraulic flow the pressure is hydrostatic, these can be written as

$$\int_{-\ell}^{\ell} (\sigma + \beta)\, B dX = 0 \tag{4}$$

and

$$\int_{-\ell}^{\ell} (\sigma + \beta)\, XB dX = 0 \tag{5}$$

(3),(4) and (5) can now be solved for s and θ, given that $\sigma = s + (X/h)\tan\theta$.

For comparison we will also be solving for sinkage and trim using the "fixed-ship" method, as used in linearized theory and as an approximation in nonlinear hydraulic theory [3]. This is done by setting $\sigma = 0$ in (3) to find $\beta(X)$, and then solving (4) and (5) simultaneously for s and θ.

4 Squat of a Fore-aft Symmetric Ship

For a fore-aft symmetric ship, the hydraulic theory predicts zero trim angle ($\theta = 0$) whenever a steady solution exists. σ is therefore constant in X, and we only require the force balance equation (4) in addition to the cubic (3) for $\beta(X)$. Since we cannot solve (3) and (4) explicitly for σ, we consider the scaled nett vertical force function

$$f(\sigma) = \int_{-\ell}^{\ell} (\sigma + \beta)\, B dX \tag{6}$$

which we require to be zero. Guessing a value for σ gives the corresponding profile of $\beta(X)$ by solving (3) at every position X. This is then used in (6) to find $f(\sigma)$. If we do this with two different guesses $\sigma = \sigma_0, \sigma_1$ we can use the secant method [4] to find a better estimate $\sigma = \sigma_2$, namely

$$\sigma_2 = \frac{\sigma_1 f(\sigma_0) - \sigma_0 f(\sigma_1)}{f(\sigma_0) - f(\sigma_1)} \tag{7}$$

Discarding whichever of the previous estimates σ_0 and σ_1 is further away from the new estimate σ_2, we can then continue the process until the converged solution for σ and corresponding profile of $\beta(X)$ are found.

4.1 Example for a Wigley Hull

Let us consider a Wigley hull [5] of length 200 metres, maximum beam $B_{max} = 40$ metres and constant draught $T = 9$ metres. It is travelling in a channel of width 100 metres and depth 12 metres. Therefore in this situation we have $B_{max}/w = 0.4$, with a maximum blockage coefficient $S_{max}/S_0 = 0.2$.

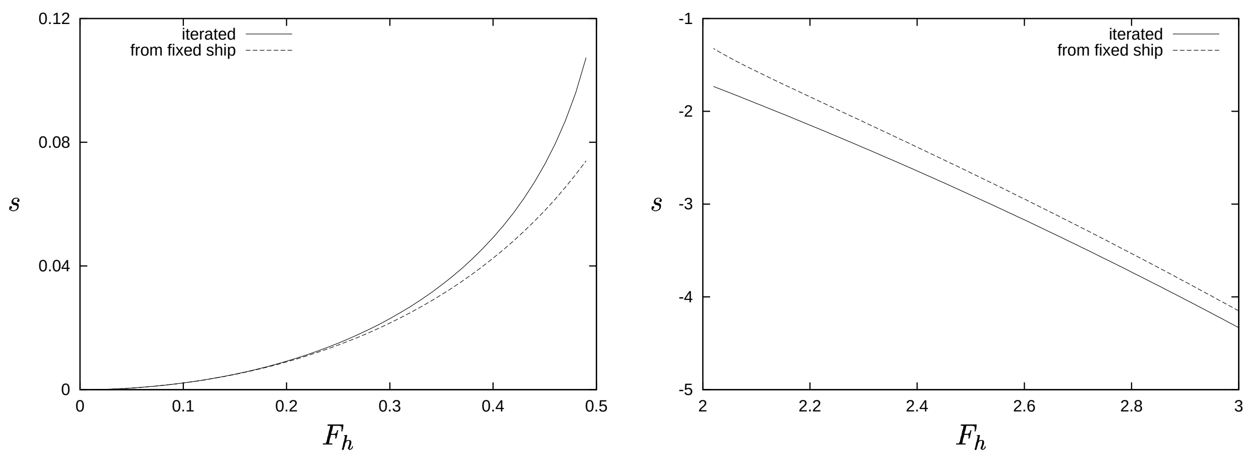

Figure 2: Scaled sinkage relative to water depth, as a function of Froude number, for a Wigley hull with $B_{max}/w = 0.4$, $S/S_0 = 0.2$. (Left) subcritical, (right) supercritical.

Fig. 2 gives the scaled sinkage, relative to water depth h, of the Wigley hull for a range of subcritical and supercritical Froude numbers. This is found using the iterating method just described, as well as the fixed-ship method for comparison.

We can see that the fixed-ship method provides a good approximation when F_h is small. This is to be expected because β and σ are also small in this case and linearization is justified.

However, for larger subcritical F_h, there is a marked increase in the sinkage from that predicted using the fixed-ship method. This is because when the ship sinks downwards, the blockage is effectively increased. For large subcritical sinkages, this effect becomes increasingly apparent.

For example, we see that when $F_h = 0.38$ (corresponding to a ship speed of 4.1 m/s or 8.0 knots) we have a scaled sinkage s of 0.0423, compared to 0.0374 for the fixed-ship method. This corresponds to an actual sinkage of 0.51 metres, compared to 0.45 metres as predicted by the fixed-ship method.

When $F_h = 0.49$ (corresponding to a ship speed of 5.3 m/s or 10.3 knots) the sinkage is 1.28 metres, compared to only 0.89 metres as predicted by the fixed-ship method.

Supercritical results for the same ship and channel configuration are also shown in Fig. 2. For supercritical flow, the sinkage is negative, corresponding to an elevation of the ship. Since steady supercritical flow in this case only exists for the unrealistic range $F_h > 2.02$ (corresponding to $U > 21.9$ m/s or 42.6 knots), the elevations are extremely large. However, we notice that the fixed-ship method again underestimates the magnitude of the sinkage.

5 Squat of a General Ship

For a general non-fore-aft-symmetric ship, we use a two-variable secant method to solve simultaneously for the quantities s and θ, using (3),(4) and (5). Only subcritical results are shown here, as steady supercritical flow can not exist at practical speeds for the ship and channel considered.

5.1 Example for a Marad Hull

We consider a Marad P-Series hull of the same length, draught and maximum beam as the Wigley hull described in section 4.1, travelling down the same channel. This ship is much more block-like, so the maximum blockage coefficient is $S_{max}/S_0 = 0.297$.

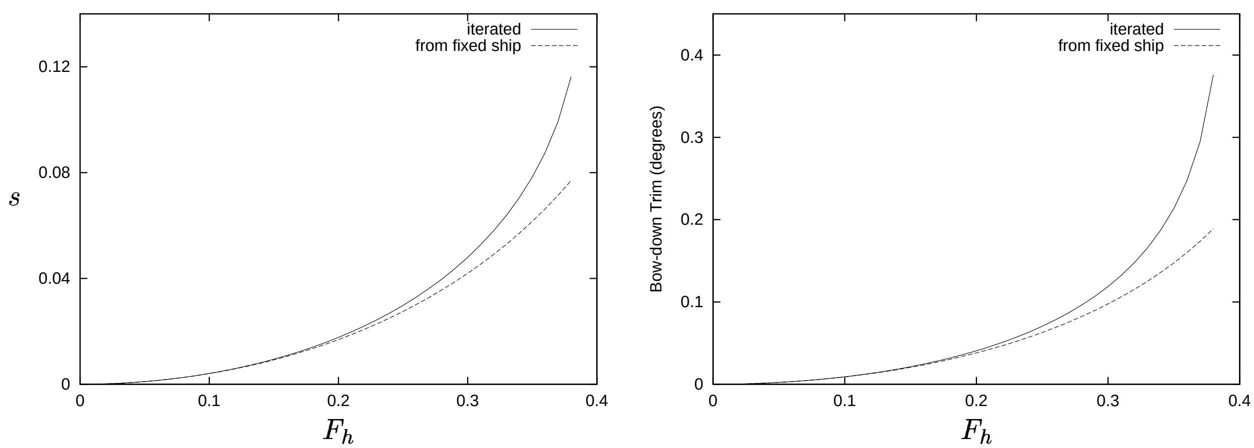

Figure 3: Scaled midships sinkage (left) and bow-down trim (right) for a Marad hull, with $B_{max}/w = 0.4$, $S_{max}/S_0 = 0.297$, $2\ell/h = 200/12$

Fig. 3 shows the scaled midship sinkage and bow-down trim of the Marad hull, using both the iterating method and the fixed-ship method. We can see again that the fixed-ship method greatly underestimates the sinkage for larger Froude numbers. Similarly the trim, which is always bow-down, increases markedly from the fixed-ship prediction when we allow the ship to squat.

From the graphs we see that when $F_h = 0.38$ (corresponding to a ship speed of 4.1 m/s or 8.0 knots) we have a scaled midship sinkage s of 0.116 and bow-down trim angle of 0.376 degrees. This gives a stern sinkage of 0.736 metres and bow sinkage of 2.05 metres for this ship, compared to only 0.594 metres and 1.25 metres respectively using the fixed-ship method.

6 Existence of Steady Flow

The system of equations (3),(4) and (5) is not always solvable. In fact, for any particular ship and channel configuration, there exists a "critical" range of F_h for which a solution to (3),(4) and (5) cannot be found for all X. This range normally extends either side of the linearized critical value $F_h = 1.0$. We will denote its boundaries as the "limiting Froude numbers" F_{lim}^- and F_{lim}^+, such that no steady hydraulic flow exists when F_h is in the range $F_{lim}^- < F_h < F_{lim}^+$.

In this critical range, there is no admissible steady hydraulic flow for which the continuity and Bernoulli conditions can be satisfied simultaneously. Physically, continuity past the ship cannot be satisfied without an increase in the energy of the system. This leads to a piling up of water at the bow of the ship and, according to hydraulic theory, a shelf of water radiating away ahead of the ship and a trough behind [2]. This is a simplified model of the actual process, which involves almost periodic production of solitary waves ahead of the ship (see e.g. [9]).

Like the sinkage and trim, the values of F_{lim}^- and F_{lim}^+ change when we allow the ship to squat.

6.1 Fixed Ship

For a fixed ship, we need only consider (3), with σ set to zero, to find the limiting Froude numbers. This method has been briefly outlined in [1].

Specifically, if we set $\sigma = 0$ in (3), for any given values of S and B we have a cubic equation for β with one, two or three real roots. This cubic has three real roots when F_h is much greater than or less than one, and only one real root (which is inadmissible) in the intermediate range denoted $F^- < F_h < F^+$. The local limiting Froude numbers F^- and F^+ are therefore the values of F_h for which (3) has exactly two solutions. This occurs when the graph of the cubic (3) becomes tangent to the β axis. The roots $F_h = F^-$,F^+ can be shown to be the solutions of

$$3\left[F_h^2\left(1-\frac{B}{w}\right)\right]^{1/3} - F_h^2\left(1-\frac{B}{w}\right) = 2\left(1-\frac{S}{S_0}\right) \tag{8}$$

Every station X, having different values of S and B, can be considered to have its own limiting Froude numbers $F^-(X)$,$F^+(X)$. However, for a steady flow to exist, it must exist along the entire length of the ship. Therefore, the actual limiting Froude numbers for a particular ship are given by

$$\begin{aligned} F_{lim}^- &= \min_X F^-(X) \\ F_{lim}^+ &= \max_X F^+(X) \end{aligned} \tag{9}$$

At this point it is worthwhile making a few observations about the local limiting Froude numbers for a fixed ship. Firstly, F^- and F^+ both move further away from unity as the blockage coefficient S/S_0 increases. This is well known. However it is not just the blockage coefficient that determines whether a steady solution exists.

For a fixed ship, any change $\Delta(\beta h)$ in the free surface height produces a change $(w - B)\Delta(\beta h)$ in $A(X)$, the local cross-sectional area taken up by the water. Therefore, if the ship's beam is comparable with the width of the channel, the free surface can drop markedly with little effect on $A(X)$. This makes for increased stability of subcritical flow, since dropping the free surface allows the fluid speed to increase (by Bernoulli's equation), so that continuity can still be satisfied.

We can see explicitly the effect of beam/channel-width ratio by writing (8) in the form

$$F^- = \frac{1}{\sqrt{1 - \frac{B}{w}}}.G^-\left(\frac{S}{S_0}\right)$$

$$F^+ = \frac{1}{\sqrt{1 - \frac{B}{w}}}.G^+\left(\frac{S}{S_0}\right) \tag{10}$$

for some functions G^-, G^+.

This shows that increasing the B/w ratio while maintaining a fixed blockage coefficient S/S_0 will increase both F^- and F^+ by the factor $1/\sqrt{1 - \frac{B}{w}}$. In particular, F^- decreases with S/S_0 but increases with B/w.

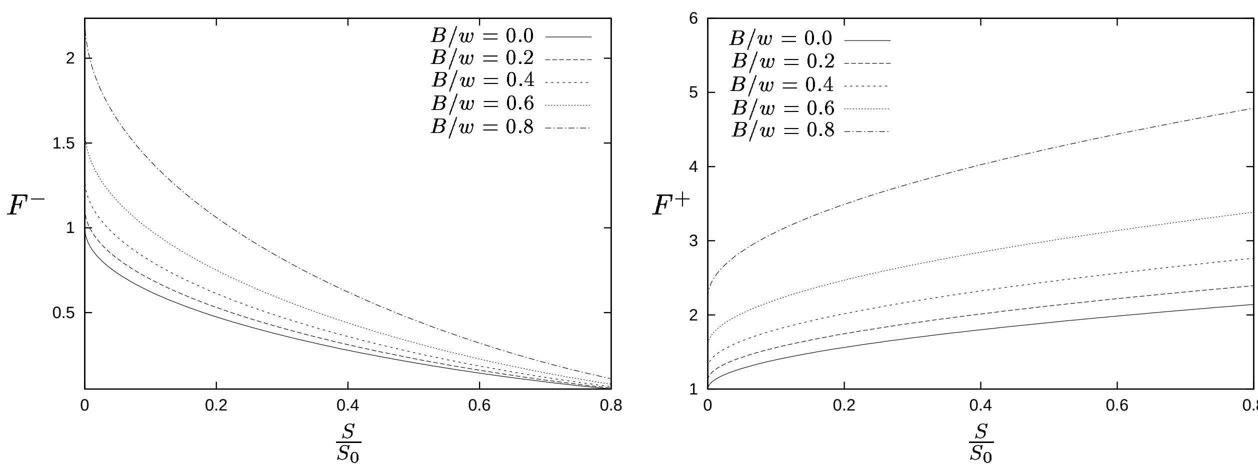

Figure 4: Local Froude number limits F^-,F^+ for steady flow around a fixed ship, as a function of blockage coefficient S/S_0 and beam/channel-width ratio B/w

In Fig. 4 we have solved (8) numerically for a range of blockage coefficients and beam/channel-width ratios. Here we see clearly how F^- and F^+ increase with the beam/channel-width ratio. In fact, for sections with small blockage coefficients but large beam/channel-width ratios (e.g. a section of small draught but large beam), F^- may be greater than one.

For a block-like ship, it is actually possible to have $F^- > 1$ along the entire ship, so that $F^-_{lim} > 1$. Hence, a fixed block-like ship of small draught and large beam can sustain steady flow, according to hydraulic theory, at Froude numbers up to and above $F_h = 1$. This is, however, impossible; the hydraulic theory is not valid in this case, as the assumptions for one-dimensional flow are violated at a blunt bow or stern.

Because of the contrasting dependence of F^- on S and B, the critical section of the ship, where $F^-_{lim} = F^-$, need not necessarily be the section of largest area. It could occur anywhere along the ship, even at the ends.

For example, a section with small blockage coefficient but large beam/channel-width ratio can have $F^- > 1$, as mentioned previously. If the ship has a sharp bow or stern, it must have $F^- = 1$ at the ends (because $F^- \to 1$ as $S, B \to 0$). Therefore in this case the ship sustains steady flow at Froude numbers up to $F_h = 1$, when the ends dictate the beginning of unsteady flow.

One more property of (8) is that the point on the ship where F^- is minimized will in general not be the same point where F^+ is maximized. That is, the critical section that determines F^-_{lim} will not always be the same critical section that determines F^+_{lim}.

6.2 Block Ship Free to Squat

Constantine [2] considered an exactly block-like ship, of constant section area and waterline beam, that is free to squat. In this case the sinkage of the ship is equal to the depression of the free surface, and the ship always maintains a constant submerged cross-section. Because of this, the ship's beam has no effect on the hydrodynamics, unlike the case of a fixed ship.

Therefore the limiting Froude numbers depend only on the blockage coefficient, and can be shown [8] to satisfy

$$3F_h^{2/3} - F_h^2 = 2\left(1 - \frac{S}{S_0}\right) \tag{11}$$

6.3 General Ship Free to Squat

As is the case for a fixed ship, the limiting Froude numbers for a ship free to squat are those values of F_h for which the graph of the cubic (3) becomes tangent to the β axis. This gives the relation

$$3\left[F_h^2\left(1 - \frac{B}{w}\right)\right]^{1/3} - F_h^2\left(1 - \frac{B}{w}\right) = 2\left(1 - \frac{S}{S_0} - \sigma\frac{B}{w}\right) \tag{12}$$

The problem is now non-local, and complicated by the fact that σ must be determined as part of the problem.

For simplicity we shall consider a fore-aft symmetric ship, where the maximum values of S and B occur at the midsection. We shall also only consider hull forms for which the midsection is the critical section. Therefore the values of S and B used in (12) are those of the midsection. The more general case, where the critical section might occur anywhere along the ship's length, will not be considered here.

Because the problem is non-local, we do not use $F^-(X)$ and $F^+(X)$; only the overall limiting Froude numbers F^-_{lim} and F^+_{lim} have meaning in this case. As each of these quantities should be determined separately, we shall specifically discuss F^-_{lim}, with the method of solution being equivalent for F^+_{lim}.

Solving (12) for F_h gives the limiting Froude number F^-_{lim} in terms of σ. However, σ must also satisfy the force balance equation (4), where $\beta(X)$ is determined through the cubic (3).

These three simultaneous equations allow F_{lim}^{-} to be determined by the secant method. Guessing a value for σ, (12) gives the corresponding value of F_h which makes the midsection on the edge of critical flow. We now know that a solution for β can be found for all X using (3). This then allows us to calculate $f(\sigma)$ as in (6), and if we do this for two initial guesses σ_0 and σ_1, we will find a better estimate σ_2 using equation (7). In this way the limiting value of σ and the corresponding F_{lim}^{-} can be found.

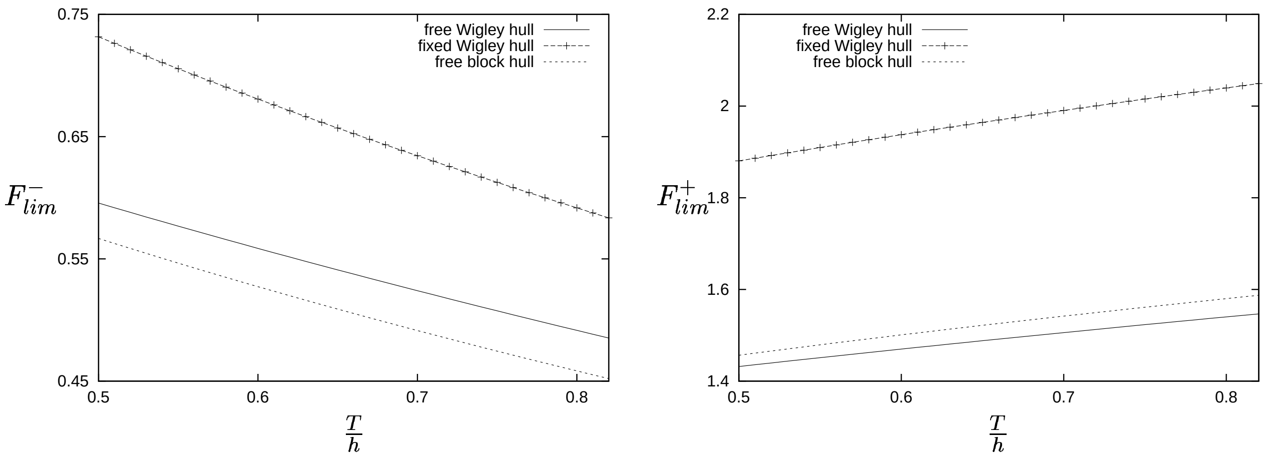

Figure 5: Limiting Froude numbers F_{lim}^{-},F_{lim}^{+} for steady flow as a function of draught/depth ratio, for a Wigley hull with $B/w = 0.4$. Results are independent of ship length.

In Fig. 5 we have plotted the limiting Froude numbers F_{lim}^{-},F_{lim}^{+} for a Wigley hull with varying draught/depth ratios T/h. This could represent ships with varying draughts in the same channel, or the same ship in channels of varying depth. The range of T/h was chosen high enough ($T/h > 0.5$) to ensure that the midsection is the critical section. Results were not given for $T/h > 0.82$ because, in the subcritical case, the ship was found to touch the channel bottom before the critical Froude number could be reached.

We can see that F_{lim}^{-} for a ship allowed to squat is significantly smaller than for the same ship held fixed. For example, when $T/h = 0.6$, the fixed Wigley hull considered here can sustain steady subcritical flow up to $F_h = 0.68$, whereas the same ship allowed to squat can only sustain steady flow up to $F_h = 0.56$. The reason for this is that allowing the ship to squat effectively increases the blockage, making subcritical flow become unsteady at a lower Froude number.

The limiting Froude numbers of the block-like hull treated by Constantine [2] have also been included for comparison. This hull has a constant cross-section which is the same as the midsection of the Wigley hull. Because it has larger section area towards the bow and stern, its sinkage is larger, causing the midsection flow to become unsteady at a lower Froude number. This clearly shows that it is not just the characteristics of the critical section that determine the limiting Froude number.

In the supercritical case, the vessel rises in the water rather than sinking. This decreases the blockage factor, as compared to the fixed ship, meaning that F_{lim}^{+} for the vessel allowed to squat is closer to unity than for the fixed ship. For example, when $T/h = 0.6$, the fixed Wigley hull considered here can only sustain steady supercritical flow for $F_h > 1.94$; the same ship allowed to rise in the water will sustain steady flow for $F_h > 1.47$.

The block-like hull rises in the water less than the Wigley hull, resulting in a larger blockage coefficient. This gives a value of F_{lim}^{+} that is further away from unity than for the free Wigley hull.

7 Conclusions

We have outlined a consistent nonlinear method for computing the flow around a ship that is free to squat, travelling in a uniform channel. In the subcritical speed range, squat increases the blockage coefficient of the ship in the channel, leading to a greater sinkage than that predicted using a fixed-ship theory.

The discrepancy increases when the depth Froude number approaches the upper limit of steady subcritical flow, as this is when the sinkage is large. In this case it is no longer acceptable to use a theory in which the ship is considered fixed vertically.

Allowing the ship to squat also causes the Froude number limit of steady subcritical flow to decrease significantly, so that critical flow begins at a lower Froude number.

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